Online supplementary material for "Personalised drawdown strategies and partial annuitisation to mitigate longevity risk"

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1 Dynamics of wealth

The retirement wealth in this study is the sum of account-based pension, savings and other financial assets. We are interested in the drawdown of these savings so consider that there is no additional contirbutions to the savings or earnings from wages. Let W_t be the total wealth at time t, then the dynamics of the wealth is as follows

$$W_{t+1} = (W_t + P_t + A_t - C_t - F_t)\, e^{R_t - q_t},$$

where P_t is the Age Pension entitlement, A_t is the annuity payment, C_t is the retirement income which is the consumption and F_t is the total fund management fees depending on balance, the fund level and the selected investment strategies at time $t = 0, 1, ...T$, where T is the number of years after retirement. The portfolio return R_t is as in Table 1, and the retirement wealth is in today's value, so is discounted by the inflation factor q_t. Both R_t and q_t are simulated by the SUPA model[1] (Chen et al., 2020). In our example later, we assume growth assets of 50% in the portfolio. When the annuitisation percentage is $\alpha \in [0, 0.5]$, the percentage of the growth asset of the non-annuitised component is $0.5/(1-\alpha)$.

Table 1: A diversified asset allocation: a strategy with 50% of *Growth* asset and 50% of Defensive' asset

Australian Equity	25%	Australian Fixed Income	25%
International Equity	15%	Cash	15%
Property	10%	International Fixed Income	10%
Growth asset	50%	Defensive asset	50%

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[1]For details of SUPA model, see: `http://risklab1.it.csiro.au:5000/SUPA-model`

We use the SUPA simulation to analyse different pre-determined drawdown strategies. In our case study, we assume a subject with personal and financial details as in Table 2. We model outcomes from the beginning of retirement, assumed to be aged 67 ($t = 0$) in line with the access age to the Age Pension from 2023, and end in the age of 104 ($T = 37$) for plotting purpose. The females have longer life expectancy than the males, therefore when using mortality related metrics to measure the drawdown strategies, the scores are different between males and females.

Table 2: User information

age	67	financial assets	A\$50,000
gender	male	other testables	A\$0
status	single	homeowner	yes

2 Drawdown Strategies

We also assume he owns A\$50,000 financial assets with no post-retirement income after retirement apart from any Age Pension entitlement, and superannuation (including any lifetime annuity purchased with superannuation savings). The investment portfolio for his superannuation fund is given in Table 1. The drawdown strategies are:

- predetermined drawdown strategies:
  - 1) minimum drawdown rate mandated by superannuation regulation: Minimum drawdown is age dependent and the annual percentages are presented in Table 3.

Table 3: Minimum drawdown rule

Age	under 65	65-74	75-79	80-84	85-89	90-94	over 95
Min DD rate (%)	4	5	6	7	9	11	14

    2) minimum rate + 1% drawdown rule:

    3) 4% of the initial balance in real term, regardless of the market performance and age.

    4) Rule of Thumbs by De Ravin et al. (2019): The baseline drawdown rate is the first digit of the age, add 2% if the $ABP \in [250000, 500000)$ indexed with CPI, subject to minimum drawdown rule.
  - targeted income drawdown strategies: 1) ASFA modest and comfortable lifestyle : A$\$27,913$ and A$\$43,787$ for single; 2) Other targets for comparison.

We select a few groups of comparison to look at the retirement income, e.g. to compare the different drawdown strategies. We will have to fix the superannuation balance, because the optimal strategy will rely on the initial balance. The annuities in this study are guaranteed income streams so the payments are known with certainty. New products coming to market in Australia do not have a guarantee so the payment levels can change, but otherwise the positioning would be similar.

3 Means-tested Age Pension

After reaching the eligibility age[1], retirees can receive the government funded Age Pension which is means-tested and determined by the current asset values (total testable assets), income, and home ownership status. At time t, P_t represents the Age Pension payment, which is the minimum value under the entitlement P_t^A after asset test and entitlement P_t^I after income test. Pensions have income and asset limits, if the assets or/and the income are over the limit, the pension will be reduced at a taper rate. The variables related to the means test are given in Table 4.

Table 4: Variables and parameters for means test for a single home owner (effective from July 2019)

Notations	Variables at time t	Notations		Value
P_t^A	asset test pension entitlement	$\hat{P}_t$	full pension including supplements	A\$24,335 p.a.
P_t^I	income test pension entitlement	$\tilde{b}_t$	asset test threshold	A\$263,250
b_t	total assessable asset	$\tilde{I}_t$	income test limit	A\$174 p.f.
I_t	total income	$\tilde{b}_t^I$	lower deeming threshold	A\$51,800
b_t^{ABP}	value of account-based pension	τ^A	taper rate for asset test	0.3%/A\$
$b_t^{annuity}$	purchase price of annuity	τ^I	taper rate for income test	0.4/A\$
b_t^{fin}	value of financial asset	r_1	lower deemed rate	1%
I_t^{deemed}	deemed income	r_2	higher deemed rate	3%
I_t^a	annuity payment	τ_{x+t}^a	annuity asset deemed rate	0.6 or 0.3
I_t^{other}	other income	τ_{x+t}^i	annuity income deemed rate	0.6

The maximum Age Pension including supplementary per fortnight is A\$933.4, that is equivalent to A\$24,335. The maximum Age Pension $\hat{P}_t$, asset test threshold $\tilde{b}_t$, income test limit $\tilde{I}_t$, lower deemed threshold $\tilde{b}_t^I$, annuity value $b_t^{annuity}$ are indexed annually with CPI. The taper rates and deeming rates are fixed to be constant based on the current policy. The total assessable asset b_t which is the sum of superannuation, deemed annuity and non-superannuation asset (financial asset and other testable asset e.g. investment property; gifting etc.) The value of the residential home is excluded. So we have $b_t = b_t^{ABP} + b_t^a + b_t^{fin}$ where $b_t^a = \tau_{x+t}^a b_t^{annuity}$ is the tapered annuity asset and $\tau_{x+t}^a = 0.6$ when $x + t < 84$ otherwise $\tau_{x+t}^a = 0.3$. The total testable income I_t is the sum of all income sources such as the deemed income from the financial assets, tapered annuity payment and other income. $I_t = I_t^{deemed} + I_t^a + I_t^{other}$ where $I_t^{deemed} = r_1 \min\left(b_t, \tilde{b}_t^I\right) + r_2 \max\left(b_t - \tilde{b}_t^I, 0\right)$, $I_t^a = \tau_{x+t}^i I_t^{annuity}$ and $\tau_{x+t}^i = 0.6$.

The asset test entitlement is the full Age Pension minus the tapered total testable asset exceeding the asset test threshold.

[1]The current Age Pension eligibility age is 66 years in July 2019, it applies to both men and women.

$$P_t^A = \max\left(\hat{P}_t - \tau^A \max\left(b_t - \tilde{b}_t\right), 0\right). \tag{1}$$

Passing the Age Pension income test is another requirement. The income test entitlement equals the full Age Pension reduceds by the tapered total income above the income test theshold

$$P_t^I = \max\left(\hat{P}_t - \tau^I \max\left(I_t - \tilde{I}_t, 0\right), 0\right). \tag{2}$$

And the Age Pension entitlement is the minimum of these two:

$$P_t = \min\left(P_t^A, P_t^I\right). \tag{3}$$

From 1 July 2019, new means test rules were being introduced to ensure the pooling of income streams such as lifetime superannuation pensions, lifetime annuity and deferred lifetime annuity. Under the new rule, 60% of all payments from pooled lifetime income stream will be counted as income and 60% of nominal purchase price counted to age 84 (or a minimum of 5 years) and 30% thereafter for life.[2] From 1 July 2019, pensioners over Age Pension age accrue any unused part of the A$300 fornightly work bonus exemption amount in a Work Bonus Income Bank[3]. The maximum work bonus income bank amount that can accrued is A$7,800. The income bank amount offsets future income from work that would otherwise be assessable under the pension income test. In this paper, we do not consider this work income exemption for simplicity.

4 Metrics and Comparisons

There are a wide range of metrics used to compare retirement income strategies from the academic literature and industry practice Callil et al. (2018). The basic metrics include the probability of ruin and compute the average age at ruin, probability of income inadequacy and the duration and depth of income misses. Metrics can be classified depending on whether they assess the strategies against a particular goal (target income), and whether the metric makes any allowance for liquidity during retirement or a bequest upon death. The outcomes in early years should count more towards the total than in later years when smaller survival probability is expected. These approaches provide a single value which can be used to compare outcomes for different withdrawal strategies and to assess of the overall effect. In this paper, we use Member's default utility function (MDUF) and score developed by Bell et al. (2017a,b); De Ravin et al. (2019) to compare these drawdown strategies. Our base case study: A single 67-year-old male homeowner with 500,000 superannuation.

[2] `https://www.dss.gov.au/seniors-budget-measures/means-test-rules-for-pooled-lifetime-income-streams`

[3] Available online at: https://www.dss.gov.au/seniors/programmes-services/work-bonus

4.1 MDUF Expected Utility

And there is a class of utility-based metrics, such as the Risk-adjusted Income and MDUF Score Bell et al. (2017a); De Ravin et al. (2019). We consider the MDUF which is a weighted sum of consumption and unspent balance, with conditional survival and mortalities as weights. At time $t = 0$ MDUF is defined as:

$$U_0 = \max_{\{c_t\}_{0 \le t \le T}} \mathbb{E}\left[\sum_{t=0}^{T} \left\{ {}_t p_x u(c_t) + {}_{t-1|}\, q_x v(b_t) \right\}\right] \tag{4}$$

subject to

$$b_t > c_t > 0,$$

where

- x is the retirement age. T is the end of time horizon, the number of years before the maximum lifespan 110. $x + T = 110$.
- c_t is the annual consumption at time t discounted by inflation back to time $t = 0$.
- b_t is the total wealth at time t. Both c_t and b_t are discounted by inflation q back to time 0.
- $u(c_t)$ is the CRRA utility function with risk averse parameter ρ ($\rho = 0$ risk neutral) defined as
$$u(c_t) = \frac{c_t^{1-\rho}}{1-\rho}. \tag{5}$$
- $v(b_t)$ is the utility function defined over end-of-life residual benefit b_t if the person dies between $t-1$ and t, with parameter ϕ which is the strength of the bequest motive. Higher ϕ means stronger residual benefit motive:
$$\begin{aligned} v(b_t) &= \frac{b_t^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho} \\ &= u(b_t) \cdot A(\phi, \rho). \end{aligned} \tag{6}$$
- ${}_t p_x$ is the probability of being alive at age $x + t$ conditional on being alive at age x.
- ${}_{t-1|}q_x v$ is the probability of dying during the period $(x + t - 1, x + t]$ conditional on being alive at age x.

To perform quantitative comparisons, we use the MDUF utility-based scores. MDUF Score is a monotonic transformation of the expected utility. It is the constant level of income (considering the trade-off against residual benefit), and is calculated as follows:

$$S_{MDUF} = \left[U_0 \times \frac{1-\rho}{\Sigma_{t=0}^{T} \left\{ {}_t p_x + {}_{t-1|}\, q_x \frac{\phi}{1-\phi} \right\}} \right]^{\frac{1}{1-\rho}}.$$

where U_0 is the expected utility in Eq. 4. The derivation is given in the technical note of Bell et al. (2017b).

We compare the following different withdrawal strategies with different annuity purchases in Table 5 and 6 with different bequest motives; $\phi = 0.5$, $A(\phi, \rho) = 1$ in Equation 6, so $v(b_t) = u(b_t)$, where same amount of consumption and bequest have equal utility. If we ignore the bequest value and focus only on the consumption utility, then $\phi = 0$. We test the risk averse level $\rho = 5$ with different bequest motives. Since the RoT is based on the parameters $\rho = 5$ and $\phi = 0$, we use the same values to compute the the expected utility and MDUF scores and compare with other drawdown strategies. We find that the comfortable target drawdown can realise similar and slightly better expected utilities and scores.

Table 5: Expected Utility (EU) and MDUF Scores ($\rho = 5$, $\phi = 0$) with three 0%, 30% , 50% lifetime annuity, and 10%, 20% DLA.

Drawdown	0 annuity		30% annuity		50% annuity		10% DLA		20% DLA	
	EU	S_{MDUF}	EU	S_{MDUF}	EU	S_{MDUF}	EU	S_{MDUF}	EU	S_{MDUF}
min	-4.77e-18	30773	-2.87e-18	34937	-2.14e-18	37585	-3.99e-18	32188	-3.37e-18	33577
min + 1%	-2.86e-18	34975	-2.04e-18	38073	-1.66e-18	40073	-2.54e-18	36033	-2.27e-18	37062
4%	-9.58e-18	25850	-6.35e-18	28647	-5.84e-18	29256	-8.03e-18	27017	-7.00e-18	27964
RoT	-1.98e-18	36853	-1.45e-18	41470	-1.37e-18	42024	-1.63e-18	40249	-1.52e-18	40943
Modest	-7.63e-18	27368	-7.63e-18	27369	-7.60e-18	27391	-7.63e-18	27368	-7.63e-18	27368
32,000	-4.08e-18	32000	-4.08e-18	32000	-4.08e-18	32004	-4.08e-18	32000	-4.08e-18	32000
37,000	-2.30e-10	36932	-2.28e-18	36999	-2.28e-18	37000	-2.29e-18	36991	-2.28e-18	36999
Comfy	**-1.74e-18**	**39587**	**-1.32e-18**	**42448**	***-1.28e-18***	***42733***	**-1.49e-18**	**41162**	**-1.37e-18**	**42040**

Table 6: Expected Utility (EU) and MDUF Scores ($\rho = 5$, $\phi = 0.5$) with three 0%, 30% , 50% lifetime annuity, and 10%, 20% DLA.

Drawdown	0 annuity		30% annuity		50% annuity		10% DLA		20% DLA	
	EU	S_{MDUF}	EU	S_{MDUF}	EU	S_{MDUF}	EU	S_{MDUF}	EU	S_{MDUF}
min	-4.77e-18	31203	-2.87e-18	35421	-2.15e-18	38102	-3.99e-18	32636	-3.37e-18	34044
min + 1%	-2.87e-18	35439	-2.05e-18	38560	-1.67e-18	40570	-2.55e-18	36505	-2.28e-18	37542
4%	-1.60e-14	4099	-5.25e-15	5419	-2.21e-15	6725	-1.17e-14	4439	-8.03e-14	4872
RoT	**-1.99e-18**	**36853**	**-1.46e-18**	**41961**	**-1.38e-18**	**42538**	**-1.65e-18**	**40725**	**-1.54e-18**	**41425**
Modest	-7.63e-18	27753	-7.63e-18	27755	-7.602e-18	27777	-7.63e-18	27753	-7.63e-18	27753
32,000	-4.08e-18	32450	-4.08e-18	32450	-4.08e-18	32455	-4.08e-18	32450	-4.08e-18	32450
37,000	-5.82e-15	5280	-7.40e-18	27968	-2.28e-18	37520	-1.28e-15	7704	-1.40e-16	13417
Comfy	-9.63e-14	2618	-3.47e-14	3380	-7.09e-15	5026	-7.52e-14	2785	-5.42e-14	3025

In Table 5, the MDUF utilty and score shows that the optimal withdrawal strategy to consume at the ASFA comfortable standard every year when the bequest value is ignored, whereas the results from Table 6 indicate that the RoT is the optimal withdrawal strategy when the bequest value has the same utility as consumption. In both cases, the optimal annuitisation strategy is to purchase 50% lifetime annuity.

References

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