EXPLORATION AND MINING REPORT 395R

THEORY OF DIFFERENTIAL VECTOR MAGNETOMETRY - A NEW METHOD FOR REMOTE DETERMINATION OF *IN SITU* MAGNETIC PROPERTIES AND IMPROVED DRILL TARGETTING

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Executive Summary

Although it is based on classical potential theory, the mathematical development presented in this report represents a novel application. The induced magnetisation of a magnetic source is proportional to the ambient magnetic field and varies in response to natural geomagnetic variations, such as diurnal changes, storm fields and pulsations. In contrast, the remanent magnetisation is independent of changes in the ambient field. The local perturbation of the geomagnetic variations arising from a subsurface magnetic body can be determined by simultaneous monitoring of geomagnetic variations over the body and at a remote base station. Total field measurements are insufficient to determine the relative contributions of remanent and induced magnetisations to the anomaly, except in a qualitative fashion. Monitoring of all three field components at the on-anomaly and base stations, however, allows the components of the second order gradient tensor of the pseudogravitational potential to be determined. This tensor depends only on the source geometry and the measurement location and is independent of the nature (remanent or induced), magnitude or direction of the source magnetisation. The tensor is, apart from a change of sign, the external analogue of the point-function demagnetising tensor.

It is shown that the following information can be obtained from the components of this tensor *without making any assumptions about source geometry or location*:

- the Koenigsberger ratio (Q), which is the ratio of remanent magnetisation intensity to induced magnetisation intensity,
- the direction of remanence,
- the direction of total (remanent + induced) magnetisation.

This information can constrain magnetic modelling prior to drilling and remove a major source of ambiguity in magnetic interpretation. In particular, the well-known non-uniqueness of dip determination when the direction of magnetisation is unknown can be eliminated. The Q value itself constrains the geological nature of the source and the remanence direction can discriminate between sources of different ages or magnetic mineralogy. Thus the information provided by this method can substantially improve *geological* interpretation of magnetic anomalies and aid prioritisation of targets.

Furthermore, the direction to the centre of a compact source can be determined directly from diagonalisation of the tensor. Repeating the procedure at another location within the magnetic anomaly can uniquely determine the location of a compact source, prior to drilling.

An alternative approach to remote determination of *in situ* magnetic properties and source location is to dispense with a remote base station and rely on simultaneous measurement of time-varying fields and their gradients at a single location within the anomaly. This procedure has the advantage of logistical simplicity and, most significantly, greatly ameliorates the requirements for accurate orientation of vector magnetometers, but requires highly sensitive gradiometers.

A number of mathematical relationships between component anomalies, which have application to processsing and interpretation of vector magnetic surveys, are derived.

1. Definition of the tensor field A(r)

The mathematical relationships derived in this report are based on standard magnetostatic theory. Brown (1962) provides a useful treatment of the subject.

Consider a homogeneous body of volume V. If it is magnetised along the x-direction, there is a magnetic anomaly produced outside the body. At a particular point P outside the body the anomalous magnetic field **ΔB** has three components, each of which is proportional to the magnitude of the magnetisation (Fig. 1), i.e.

$$\Delta B_x = a_{xx} J_x ,$$

$$\Delta B_y = a_{yx} J_x ,$$

$$\Delta B_z = a_{zx} J_x ,$$

where the a_{ij} are constants (i.e they are independent of **J**) at each point P, but vary from place to place.

Similarly, magnetisations along the y- or z-axes produce anomalous components that are proportional to the magnetisation. By linear superposition, the components of the anomalous magnetic field can be written:

$$\Delta B_x = a_{xx} J_x + a_{xy} J_y + a_{xz} J_z ,$$

$$\Delta B_y = a_{yx} J_x + a_{yy} J_y + a_{yz} J_z ,$$

$$\Delta B_z = a_{zx} J_x + a_{zy} J_y + a_{zz} J_z .$$

These equations can be written more succinctly as:

$$\Delta B_i = \sum_j a_{ij} J_j \quad (i,j = x,y,z), \tag{1}$$

or in matrix form as:

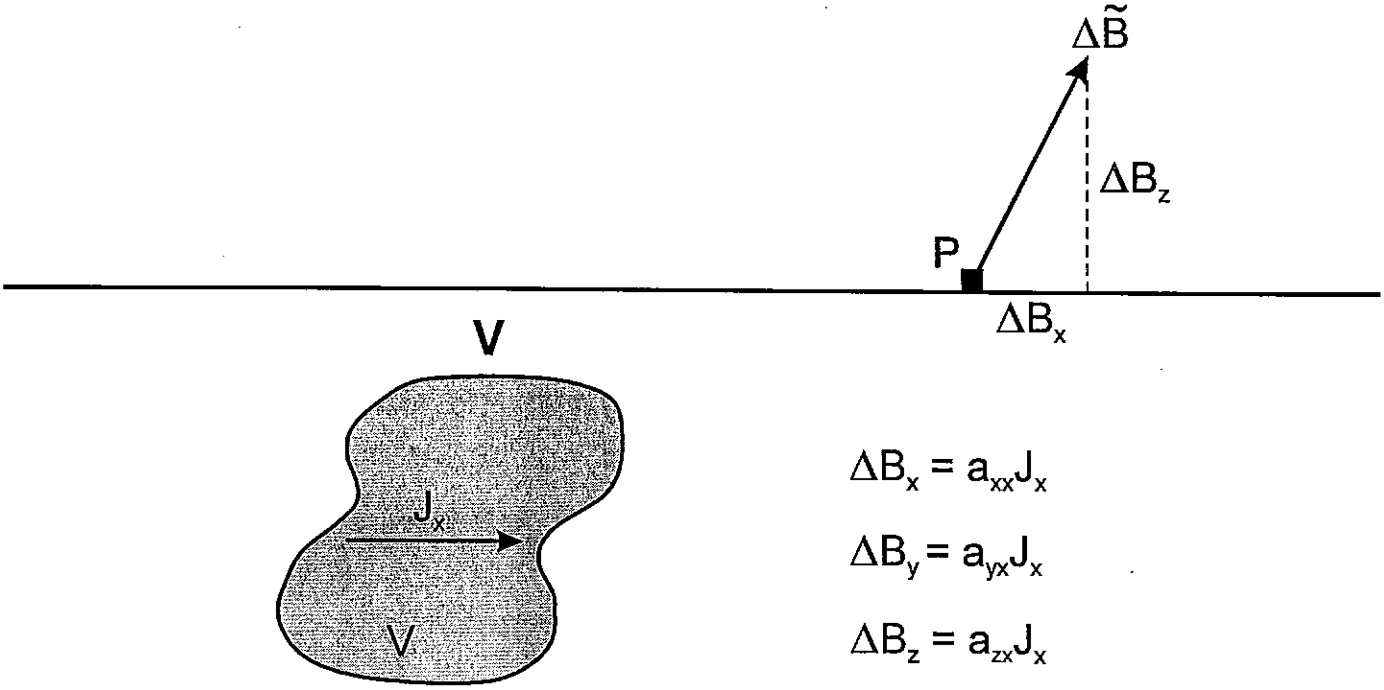
$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{bmatrix} \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} \tag{2}$$

As equation (1) shows, the coefficients a_{ij} represent a linear relationship between two vectors. This implies that the a_{ij} are components of a second order tensor **A**, with

$$\mathbf{\Delta B} = \mathbf{A} \cdot \mathbf{J} . \tag{3}$$

A(r) is a second order tensor field, which depends only on geometry, i.e. the shape, size and position of the source, and is independent of the nature (remanent or induced), magnitude or direction of the magnetisation.

DEFINITION OF TENSOR ELEMENTS a_{ij}



$$\Delta B_x = a_{xx} J_x$$

$$\Delta B_y = a_{yx} J_x$$

$$\Delta B_z = a_{zx} J_x$$

Figure 1

2. Explicit form and properties of the tensor A

In appropriate units, the pseudo-gravitational potential U of the body V is equivalent to the magnetic scalar potential that would be produced by a distribution of unit magnetic pole density throughout V. U may be expressed formally as a volume integral:

$$U(P) = \int_V \frac{dV}{r}, \tag{4}$$

where r is the distance from the volume element dV to the observation point P (see Fig.2). The magnetic scalar potential Ω due to the uniform distribution of magnetisation, **J**, throughout V is given by Poisson's relationship:

$$\Omega = -\nabla U \cdot \mathbf{J}. \tag{5}$$

The anomalous magnetic field due to the body V is therefore:

$$\Delta\mathbf{B} = -\nabla\Omega = \nabla\nabla U \cdot \mathbf{J} \tag{6}$$

Comparison of eqns (3) and (6) shows that **A** may be written explicitly as:

$$\mathbf{A} = \nabla\nabla U. \tag{7}$$

Equation (7) shows that **A** is the *second order gradient tensor of the pseudo-gravitational potential.* The components of **A** are:

$$a_{ij} = \frac{\partial^2 U}{\partial x_i \, \partial x_j}. \tag{8}$$

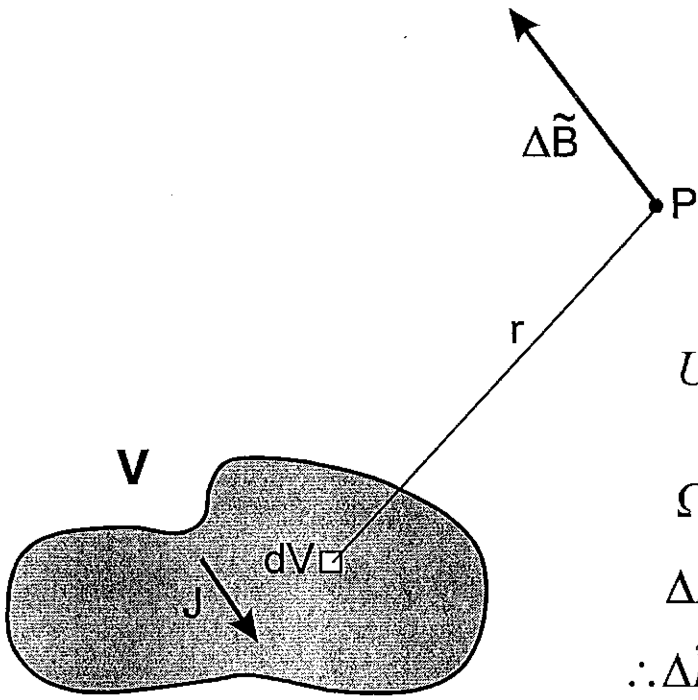
Two important properties of **A** follow from (8):

$$a_{ij} = \frac{\partial^2 U}{\partial x_i \, \partial x_j} = \frac{\partial^2 U}{\partial x_i \, \partial x_j} = a_{ij}, \tag{9}$$

$$a_{xx} + a_{yy} + a_{zz} = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = \nabla^2 U = 0. \tag{10}$$

Equation (10) follows from the fact that U is a potential field, which obeys Laplace's equation ($\nabla^2 U = 0$) outside V. Equations (9) and (10) respectively state that **A** is *symmetric* and *traceless.* The same analysis may be applied to a point within the magnetised body, with one significant difference. Inside the body U obeys Poisson's equation ($\nabla^2 U = -1$ in SI; $\nabla^2 U = -4\pi$ Oe/G in the Gaussian CGS system), rather than Laplace's equation, implying that the trace of the tensor $\mathbf{N} = -\nabla\nabla U$ is 1 SI (4π Oe/G), rather than zero. **N** is in fact the demagnetising tensor for the internal point (Brown, 1962). *Thus -**A** is the external analogue of the point-function demagnetising tensor.*

PSEUDO-GRAVITATIONAL POTENTIAL U, MAGNETIC SCALAR POTENTIAL Ω AND ANOMALOUS FIELD $\Delta\tilde{B}$



$$U = \int_V \frac{dV}{r}$$

$$\Omega = -\tilde{\nabla} U \cdot \tilde{J}$$

$$\Delta\tilde{B} = -\tilde{\nabla}\Omega = \tilde{\nabla}\tilde{\nabla} U \cdot \tilde{J}$$

$$\therefore \Delta\tilde{B} = A \cdot \tilde{J}$$

$$A = \tilde{\nabla}\tilde{\nabla} U$$

$$a_{ij} = \frac{\partial^2 U}{\partial x_i \partial x_j}$$

Figure 2

Because the matrix $[a_{ij}]$ is symmetric, it can be diagonalised by suitable rotation of co-ordinate axes. The eigenvectors $\mathbf{u_i}$ (i = 1,2,3) define three mutually orthogonal axes. With respect to this new set of Cartesian axes $[a_{ij}]$ is diagonal:

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix}. \qquad (11)$$

Because the trace of a symmetric matrix is invariant under rotation of axes, the matrix in (11) is also traceless, i.e.

$$a_1 + a_2 + a_3 = a_{11} + a_{22} + a_{33} = 0. \qquad (12)$$

In eqn (12), the $a_i = a_{ii}$ (i =1,2,3) are the eigenvalues of the matrix $[a_{ij}]$, corresponding to the eigenvectors $\mathbf{u_i}$.

Thus for every point external to the magnetic body V, *there exist three mutually orthogonal directions for which the anomalous field is coaxial and proportional to the magnetisation.* With respect to this co-ordinate system, the relationship between magnetisation and the anomalous field is particularly simple:

$$\left.\begin{aligned} \Delta B_1 &= a_1 J_1 \\ \Delta B_2 &= a_2 J_2 \\ \Delta B_3 &= a_3 J_3. \end{aligned}\right\} \qquad (13)$$

A special case arises if the source is two-dimensional. If the i-axis is parallel to the strike of the 2D body, J_i makes no contribution to the anomaly, because the poles produced at the "ends" of a body with infinite strike extent are infinitely distant. It follows that for 2D sources:

$$\left.\begin{aligned} a_1 &= 0 \\ \Delta B_1 &= 0 \\ \Delta B_2 &= a_2 J_2 \\ \Delta B_3 &= a_3 J_3. \end{aligned}\right\} \quad \text{(2D source)} \qquad (13a)$$

From (13) it follows immediately that a given direction of $\Delta\mathbf{B}$ at a given point determines the direction of $\mathbf{J}$ uniquely, provided the eigenvalues of $\mathbf{A}$ are all non-zero. If magnetisation $\mathbf{J'}$ produces an anomalous field $\Delta\mathbf{B'}$ that is parallel to $\Delta\mathbf{B}$, then $\Delta\mathbf{B'} = \lambda\Delta\mathbf{B}$, for some constant λ. Then, from (3):

$$\Delta\mathbf{B'} = \mathbf{A}\,.\,\mathbf{J'} = \lambda\,\mathbf{A}\,.\,\mathbf{J} = \mathbf{A}.(\lambda\mathbf{J}),$$

$$\therefore\ \mathbf{A}.(\mathbf{J'} - \lambda\mathbf{J}) = \mathbf{0},$$

which implies that $\mathbf{J}' = \lambda\mathbf{J}$, provided **A** is non-singular. For 3D bodies the a_i are all non-zero and det(**A**) = $a_1a_2a_3$ is also non-zero. Thus $\mathbf{J}' = \lambda\mathbf{J}$, and therefore **J'** and **J** are parallel. In other words, *parallel anomaly vectors imply parallel magnetisations*. In particular, if the anomaly components produced by remanence and induced magnetisation are parallel at any measurement point, the remanence is parallel to the induced magnetisation and is likely to be of viscous origin, rather than an ancient component. In the 2D case, one of the eigenvalues is zero and the corresponding component of magnetisation is indeterminate. From (13a), however, the direction of magnetisation, projected onto the plane perpendicular to strike, is uniquely determined by the anomaly vector components within this plane.

3. Uniformly magnetised sphere

As a specific example, consider a uniformly magnetised sphere (Fig.3). The pseudo-gravitational potential is equivalent to that of a point source at the centre, i.e. U = V/r, where r is the distance from the centre to the observation point. Applying (5), the corresponding magnetic scalar potential is:

$$\Omega = \frac{\mathbf{m}.\hat{\mathbf{r}}}{r^2}, \qquad (14)$$

where **m** = **J**V is the magnetic moment of the sphere.

By (6) and (14), the anomalous field at P is given by:

$$\Delta\mathbf{B} = \frac{-\mathbf{m} + 3(\mathbf{m}.\hat{\mathbf{r}})\hat{\mathbf{r}}}{r^3}. \qquad (15)$$

From (15), the components of **A** may be written explicitly as:

$$[a_{ij}] = V\begin{bmatrix} \frac{2x^2-y^2-z^2}{r^5} & \frac{3xy}{r^5} & \frac{3xy}{r^5} \\ \frac{3xy}{r^5} & \frac{2y^2-z^2-x^2}{r^5} & \frac{3xy}{r^5} \\ \frac{3xz}{r^5} & \frac{3yz}{r^5} & \frac{2z^2-x^2-y^2}{r^5} \end{bmatrix}, \qquad (16)$$

or more succinctly:

$$a_{ij} = \frac{3x_ix_j - r^2\delta_{ij}}{r^5}, \qquad (16a)$$

where δ_{ij} is the Kronecker delta, which has diagonal elements equal to 1 and off-diagonal elements equal to zero.

It is evident that **A** is symmetric and traceless, as required. Referring to Fig.3, it is obvious that if **J** is parallel to the radius vector **r**, then **ΔB** is parallel to **J**, and if **J** is perpendicular to **r**, then **ΔB** is antiparallel to **J**. Thus the eigenvectors of the matrix in (16) are parallel or perpendicular to **r**. In terms of the spherical polar co-ordinates (r,θ,ϕ):

$$\mathbf{u}_1 = \hat{\mathbf{r}}, \quad \mathbf{u}_2 = \hat{\theta}, \quad \mathbf{u}_3 = \hat{\phi}. \tag{17}$$

The corresponding eigenvalues are:

$$a_1 = \frac{2V}{r^3}, \quad a_2 = -\frac{V}{r^3}, \quad a_3 = -\frac{V}{r^3}. \tag{18}$$

Equation (17) implies that the direction to the centre of the sphere can be determined from the tensor **A** by diagonalisation.

4. Determination of the tensor A using differential vector variometry

The total magnetisation of subsurface magnetic sources is the vector sum of the induced and remanent magnetisations. The remanent magnetisation is constant in time, whereas the induced magnetisation varies with time due to changes in the geomagnetic field. These geomagnetic variations include diurnal variation, magnetic storm fields and pulsations. Thus the magnetisation of a source may be written as:

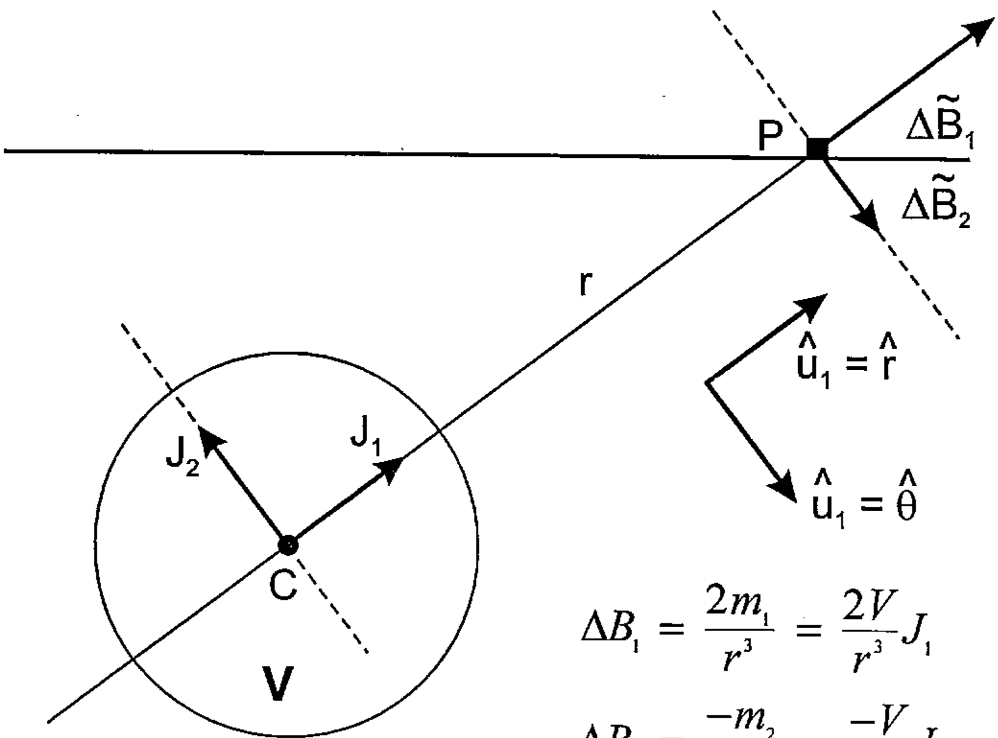
$$\mathbf{J}(t) = \mathbf{J}_I(t) + \mathbf{J}_R = k\mathbf{F}(t) + \mathbf{J}_R, \tag{19}$$

where k is the effective susceptibility, **F** is the ambient geomagnetic field, and the subscripts I and R indicate induced and remanent magnetisations respectively. Equation (19) assumes that the susceptibility is isotropic, which is a reasonable approximation for most rocks.

In the vicinity of a magnetic body the geomagnetic field is perturbed by the anomalous field, **ΔB**, which is a function of the total magnetisation **J**. Thus the magnetic anomaly itself is a function of time. At each point, **ΔB** can be expressed as the sum of a constant field, which arises from remanent magnetisation plus the induced magnetisation in the time-averaged geomagnetic field, and a time-varying component, which corresponds to the magnetisation induced by geomagnetic variations, i.e.

$$\left.\begin{aligned} \Delta B(t) &= \Delta B_o + \delta(\Delta B) = \Delta B(\mathbf{J}_o) + \Delta B(k\delta\mathbf{F}) \\ \therefore\ \Delta B(t) &= \mathbf{A}.\mathbf{J}_o + \mathbf{A}.k\delta\mathbf{F} = \mathbf{A}.(\mathbf{J}_R + k\mathbf{F}_o) + \mathbf{A}.k\delta\mathbf{F} \\ \delta(\Delta B) &= k\mathbf{A}.\delta\mathbf{F} \end{aligned}\right\} \tag{20}$$

EIGENVECTORS OF [a_{ij}] FOR A SPHERE



$$\Delta B_1 = \frac{2m_1}{r^3} = \frac{2V}{r^3}J_1$$

$$\Delta B_2 = \frac{-m_2}{r^3} = \frac{-V}{r^3}J_2$$

$$a_{11} = a_1 = \frac{2V}{r^3}$$

$$a_{22} = a_2 = -\frac{V}{r^3}$$

Figure 3

where δ is used to indicate temporal variations, while Δ indicates spatial changes. The zero subscript indicates time-averaged values or, to a good approximation, the initial values when measurements start. Figure 4 gives a schematic view of the implementation of the method.

Equation (20) shows that if $\Delta\mathbf{B}(t)$ is measured at any point P in the vicinity of the magnetic source, whilst $\delta\mathbf{F}$ is monitored simultaneously at a remote point, then k**A** at that point can be determined by matrix inversion. Because k is a scalar, the elements of the tensor **A** can be determined at P, within a multiplicative constant. The eigenvectors of **A** can therefore be uniquely defined, as can the ratios of the eigenvalues. In particular, for a quasispherical body the eigenvector corresponding to the largest eigenvalue of k**A** points away from the centre of the body.

5. Remote determination of *in situ* magnetic properties of an anomaly source

In terms of the components (ΔX, ΔY, ΔZ) of $\Delta\mathbf{B}$ and the components (X,Y,Z) of **F** eqn (20) gives:

$$\begin{bmatrix} \delta(\Delta X) \\ \delta(\Delta Y) \\ \delta(\Delta Z) \end{bmatrix} = k \begin{bmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{bmatrix} \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix} \qquad (21)$$

Using measurements of $\delta(\Delta\mathbf{B})$ and $\delta\mathbf{F}$, eqn (21) can be solved in the least squares sense for the elements ka_{ij}. The eigenvectors of $k[a_{ij}]$ define a set of axes for which $\Delta\mathbf{B}_o$ and $\mathbf{J}_o$ are coaxial. With respect to these axes:

$$(\Delta\mathbf{B}_o)_i = a_i(\mathbf{J}_o)_i = \frac{ka_i(\mathbf{J}_o)_i}{k}, \qquad (22)$$

$$\therefore \frac{\mathbf{J}_o}{k} = \left[\frac{(\Delta B_o)_1}{ka_1}, \frac{(\Delta B_o)_2}{ka_2}, \frac{(\Delta B_o)_3}{ka_3}\right]. \qquad (23)$$

Thus the direction of the time-averaged total magnetisation $\mathbf{J}_o$ can be determined from the time-averaged anomalous field $\Delta\mathbf{B}_o$ and the eigenvalues of the observed tensor k**A**. The magnitude of $\mathbf{J}_o$/k is also defined by (23), but the intensity of magnetisation is indeterminate if k is unknown.

From (19) we have:

$$\frac{\mathbf{J}_R}{k} = \frac{\mathbf{J}_o}{k} - \mathbf{F}_o. \qquad (24)$$

Equations (23) and (24) determine the direction of $\mathbf{J}_R$ and the magnitude of $\mathbf{J}_R$/k. The Koenigsberger ratio Q can also be determined using (24) because:

$$Q = \frac{|\mathbf{J}_R|}{k|\mathbf{F}|} = \frac{|\mathbf{J}_R / k|}{|\mathbf{F}|}. \qquad (25)$$

Therefore the following properties of the source can be determined from measurements of $\delta(\Delta\mathbf{B})$ and $\delta\mathbf{F}$, without making any assumptions about the source geometry:

- *the direction of the total magnetisation,*
- *the direction of the remanent magnetisation,*
- *the Koenigsberger ratio Q.*

6. Application of differential vector variometry to drill targetting

Figure 5 illustrates the application of measurements of the tensor k**A** to targetting a compact magnetic source. In this context, a compact source is one for which the dipole field is the dominant component of the multipole field arising from the source. In practice, this requires that the longest axis of the body is smaller than the the distance from the centre to the observation point P. At P the eigenvector corresponding to the positive eigenvalue of k**A** points away from the "centre of magnetisation". Thus the drilling direction from P to intersect the source is determined. If the measurements are repeated at a different point P′ that is also within the anomaly, the location of the centre of the source is uniquely defined.

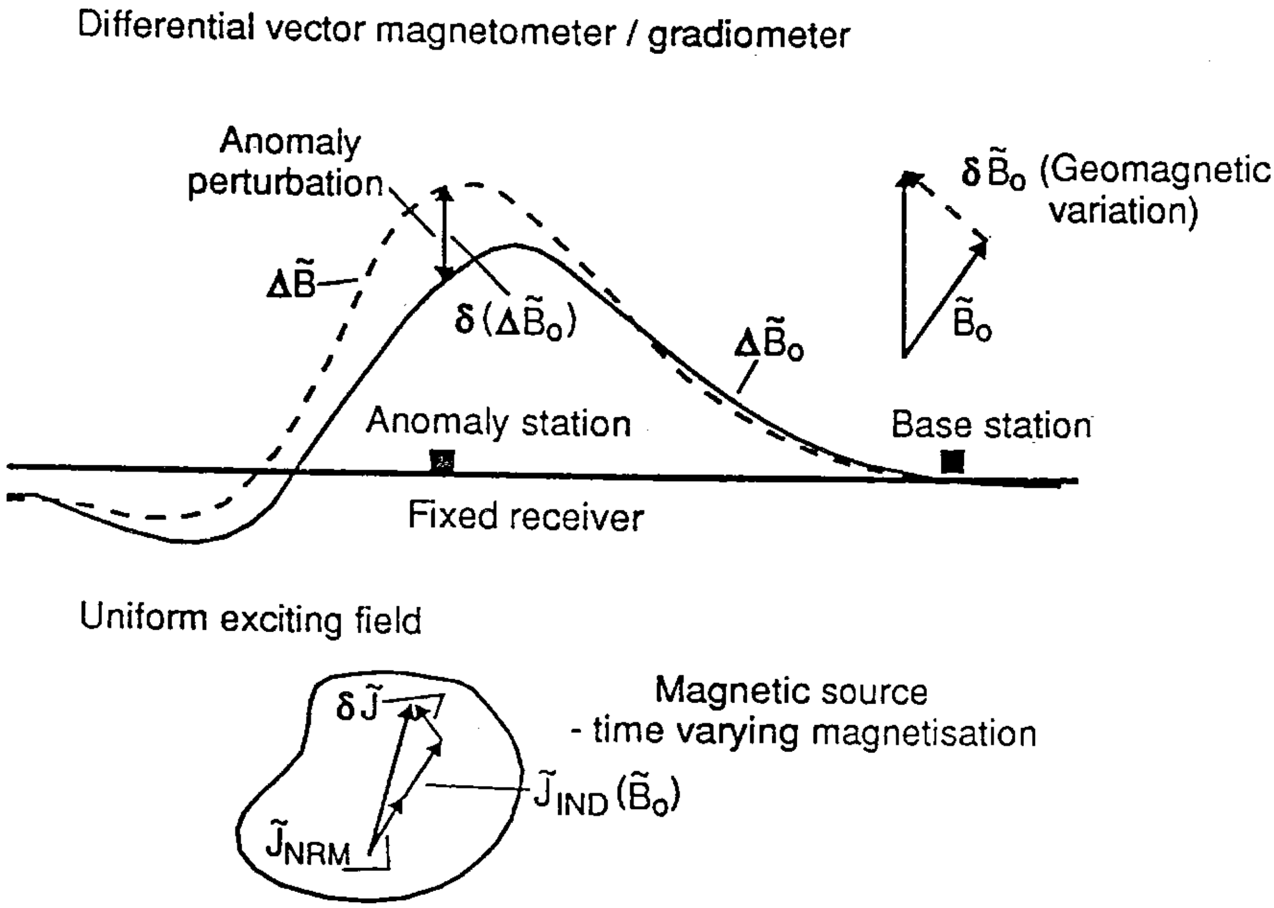

Differential vector magnetometer / gradiometer
Anomaly
perturbation
$\Delta\tilde{B}$
$\delta(\Delta\tilde{B}_o)$
$\delta\tilde{B}_o$ (Geomagnetic
variation)
$\tilde{B}_o$
$\Delta\tilde{B}_o$
Anomaly station
Base station
Fixed receiver
Uniform exciting field
$\delta\tilde{J}$
Magnetic source
- time varying magnetisation
$\tilde{J}_{IND}(\tilde{B}_o)$
$\tilde{J}_{NRM}$

Figure 4

DRILL TARGETTING FOR COMPACT SOURCE

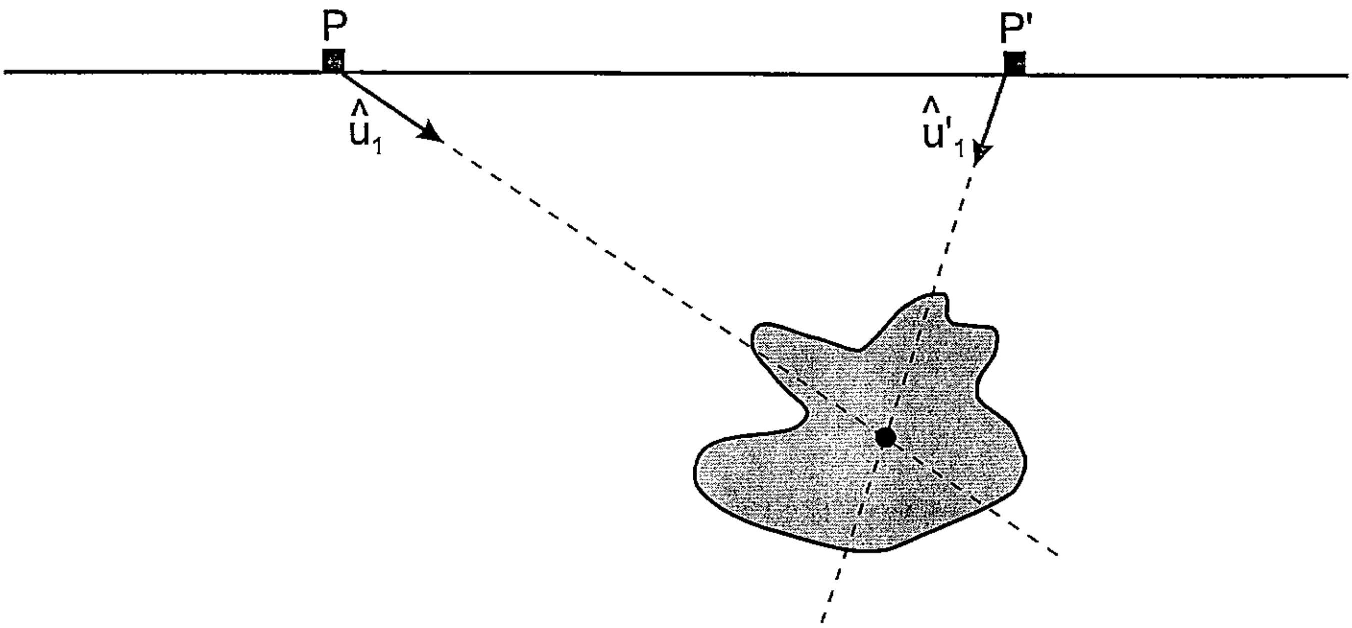


Figure 5

7. Generalisation to gradiometry

The relationship between the gradient of the anomalous field and the source magnetisation may be obtained from (3):

$$\nabla(\Delta\mathbf{B}) = \nabla\mathbf{A}\,.\,\mathbf{J}\,. \qquad (26)$$

In terms of components:

$$b_{ik} = \frac{\partial(\Delta B)_i}{\partial x_k} = \sum_j \frac{\partial a_{ij}}{\partial x_k} J_j \qquad (i,j,k = x,y,z), \qquad (27)$$

where b_{ik} is the gradient tensor of the anomalous field. The analogous relations to (20) for the gradient tensor are:

$$\left.\begin{aligned}
\nabla(\Delta\mathbf{B})[t] &= \nabla(\Delta\mathbf{B}_o) + \delta[\nabla(\Delta\mathbf{B})] = \nabla(\Delta\mathbf{B})[\mathbf{J}_o] + \nabla(\Delta\mathbf{B})[k\delta\mathbf{F}] \\
\therefore\ \nabla(\Delta\mathbf{B})[t] &= \nabla\mathbf{A}.\mathbf{J}_o + \nabla\mathbf{A}.k\delta\mathbf{F} = \nabla\mathbf{A}.(\mathbf{J}_R + k\mathbf{F}_o) + \nabla\mathbf{A}.k\delta\mathbf{F} \\
\delta[\nabla(\Delta\mathbf{B})] &= k\nabla\mathbf{A}.\delta\mathbf{F}
\end{aligned}\right\} \qquad (28)$$

The temporal variation in the components of the gradient tensor can be obtained from (20) and (27), or directly from (28):

$$\delta b_{ik} = \delta\frac{\partial(\Delta\mathbf{B})_i}{\partial x_k} = \delta\sum_j \frac{\partial a_{ij}}{\partial x_k} J_j = \sum_j \frac{\partial a_{ij}}{\partial x_k}\delta J_j = k\sum_j \frac{\partial a_{ij}}{\partial x_k}\delta\mathbf{F}_j\,. \qquad (29)$$

Equation (29) implies that simultaneous monitoring of $\nabla(\Delta\mathbf{B})$ and $\mathbf{F}$ allows $k\nabla\mathbf{A}$ to be determined. It suffices to monitor time variations for one row of the field gradient tensor, e.g. $\partial(\Delta\mathbf{B})/\partial z$, together with $\delta\mathbf{F}$ to determine $\partial\mathbf{A}/\partial z$, which is a symmetric traceless tensor. Diagonalising $\partial\mathbf{A}/\partial z$ enables $\mathbf{J}_o/k$ to be determined, in an analogous manner to that of section 6. Determination of the other properties follows directly.

8. Relationships between component anomalies

From the foregoing theory we can derive some useful relationships between anomaly components arising from particular directions of magnetisation. Denote the i-component of the anomalous field arising from the j-component of magnetisation by $\Delta B_i(J_j)$. From (3) and (9):

$$\left.\begin{aligned} \frac{\Delta B_x(J_y)}{J_y} &= a_{xy} = a_{yx} = \frac{\Delta B_y(J_x)}{J_x}, \\ \frac{\Delta B_x(J_z)}{J_z} &= a_{xz} = a_{zx} = \frac{\Delta B_z(J_x)}{J_x} \\ \frac{\Delta B_z(J_y)}{J_y} &= a_{zy} = a_{yz} = \frac{\Delta B_y(J_z)}{J_z}. \end{aligned}\right\} \qquad (30)$$

Furthermore, from (3) and (10):

$$\frac{\Delta B_x(J_x)}{J_x} + \frac{\Delta B_y(J_y)}{J_y} + \frac{\Delta B_z(J_z)}{J_z} = 0. \qquad (31)$$

Equivalent relationships to (30) and (31) have been derived, in a less direct fashion, by Affleck (1958). The relationships of (30) allow anomalies at, for example, the geomagnetic equator to be related to those at the magnetic poles. As an illustration, for an inductively magnetised source of given geometry the vertical component anomaly at the equator has identical form to the horizontal component anomaly at the pole. At any location, a remanently magnetised source, with vertical resultant magnetisation, has a horizontal component anomaly that is identical in form to the vertical component anomaly of a horizontally magnetised source of the same shape.

From (31) it follows that bodies that are symmetric about a vertical axis, inductively magnetised at the equator, have horizontal component anomalies that are half the amplitude and opposite in sign to the vertical component anomaly over an identical body at the pole. At any location, if such bodies are remanently magnetised, the horizontal component anomaly associated with, and parallel to, a horizontal resultant magnetisation is opposite in sign and half the amplitude of the vertical component anomaly associated with vertical resultant magnetisation.

Some useful relationships between component anomalies, which are applicable to processing and interpretation of vector magnetic surveys, follow directly from potential field theory. From (6):

$$\Delta B_x = -\frac{\partial \Omega}{\partial x}, \tag{32}$$

$$\therefore \frac{\partial(\Delta B_x)}{\partial y} = \frac{\partial^2 U}{\partial y \partial x} = \frac{\partial^2 U}{\partial x \partial y} = \frac{\partial(\Delta B_y)}{\partial x} \tag{33}$$

Similarly,

$$\left.\begin{aligned} \frac{\partial(\Delta B_x)}{\partial z} &= \frac{\partial(\Delta B_z)}{\partial x} \\ \frac{\partial(\Delta B_y)}{\partial z} &= \frac{\partial(\Delta B_z)}{\partial y} \\ \frac{\partial(\Delta B_z)}{\partial x} &= \frac{\partial(\Delta B_x)}{\partial z} \end{aligned}\right\}. \tag{34}$$

From (10), or directly from the Maxwell equation $\nabla.(\Delta \mathbf{B}) = 0$, it follows that:

$$\therefore \frac{\partial(\Delta B_x)}{\partial x} + \frac{\partial(\Delta B_y)}{\partial y} + \frac{\partial(\Delta B_z)}{\partial z} = 0. \tag{35}$$

Equations (31)-(35) show that spatial derivatives of anomalous field components are not independent. Measurement of some gradient components can define the *local* values of other components uniquely. This provides a tighter constraint on undersampled component data than can be provided by a single component survey (e.g. a conventional total field survey) that is used to calculate component anomalies. The local spatial derivatives of anomlous components can be used to aid interpolation between survey lines and to calculate vertical gradients. For example, if the survey traverse is along the x-axis, measurement of three component data determines the x-derivatives of $\Delta \mathbf{B}$. The transverse (y-) and vertical (z-) derivatives of ΔB_x are immediately given by (33)-(34). This information can be used to interpolate ΔB_x between lines and to produce a vertical derivative profile. If the vertical derivatives of ΔB_y and ΔB_z are measured in addition to the three components along the traverse, all local transverse and vertical derivatives are determined. This can aid gridding, particularly if the line spacing is slightly too wide to adequately sample the field. Vertical derivatives are useful for emphasising shallow features and defining geological boundaries.

9. Status of the method

The natural ambient field varies due to diurnal variation, pulsations and magnetic storm activity. In conventional magnetic surveys these variations are monitored at a base station, so that a first-order removal of their effects on the survey data can be carried out. However, the induced magnetisation of magnetic bodies varies in phase with the ambient field, producing small local perturbations of the regional geomagnetic variations. These local perturbations can be measured by monitoring geomagnetic variations at two locations simultaneously, one location within the magnetic anomaly associated with a magnetic body and the other location away from the zone of influence of the body. These effects were first observed as long ago as 1962 (Ward and Ruddock, 1962; Goldstein and Ward, 1966), using Rubidium vapour total field sensors.

More recently, Parkinson and Barnes (1985) detected the amplification of geomagnetic variations caused by the Savage River magnetite deposit, using three-axis fluxgate magnetometers. By assuming that the remanent magnetisation of this deposit is predominantly viscous remanence directed parallel to the induced magnetisation, these authors concluded that the Koenigsberger ratio of the Savage River orebody is ~0.4. A theorem, due to the present author, that parallel anomalous field vectors, resulting from remanent and induced magnetisation, imply parallelism of these magnetisation components, was required to draw this conclusion. This theorem was included as an appendix in the paper by Parkinson and Barnes (1985).

This theorem provided the basis for the further development of the theory that is presented in this report. Practical implementation of the method has involved overcoming numerous technical difficulties, some of which remain recalcitrant. A companion report (Schmidt and Clark, 1997) details the outcomes of the two year AMIRA project P446. For the first time, remote determination of *in situ* magnetic properties and source location using differential vector magnetometry have been demonstrated in the field during the course of P446. The field trials suggest that the alternative, gradiometer-magnetometer configuration that was discussed in section 7 may be the most practicable option for further development of the method. The most feasible technology for implementation of the alternative method appears to be newly developed high-temperature SQUID sensors.

Apart from research applications of this method, there are important applications to exploration. The *in situ* magnetic property information can constrain magnetic modelling prior to drilling and remove a major source of ambiguity in magnetic interpretation. In particular, the well-known non-uniqueness of dip determination when the direction of magnetisation is unknown can be eliminated, reducing the chances of drilling down-dip. The Q value itself constrains the geological nature of the source and the remanence direction can discriminate between sources of different ages or magnetic mineralogy. Thus the information provided by this method can substantially improve *geological* interpretation of magnetic anomalies and aid prioritisation of targets. Furthermore, the direction to the centre of a compact source can be determined directly from diagonalisation of the tensor. Repeating the procedure at another location within the magnetic anomaly can uniquely determine the location of a compact source, prior to drilling.

10. Conclusions

The anomalous magnetic field, $\Delta \mathbf{B}$, due to a uniformly magnetised source is linearly related to the magnetisation vector, **J**, i.e. $\Delta \mathbf{B} = \mathbf{A} \cdot \mathbf{J}$, where **A** is a second order tensor field, with nine components, which depends only on geometry, i.e. the shape, size and position of the source, and is independent of the nature (remanent or induced), magnitude or direction of the magnetisation. **A** is explicitly given by: $A = \nabla\nabla U$, where U is the pseudogravitational potential of the magnetic source. Thus **A** is the *second order gradient tensor of the pseudogravitational potential.* This tensor is, apart from a change of sign, the external analogue of the point-function demagnetising tensor. **A** is symmetric and traceless, which reduces the number of independent components to five. Furthermore, the matrix of components of **A** can be diagonalised. The eigenvecors of [A] define three mutually orthogonal directions along which $\Delta \mathbf{B}$ and **J** are coaxial.

The induced magnetisation of a magnetic source is proportional to the ambient magnetic field and varies in response to natural geomagnetic variations, such as diurnal changes, storm fields and pulsations. In contrast, the remanent magnetisation is independent of changes in the ambient field. The local perturbation of the geomagnetic variations arising from a subsurface magnetic body can be determined by simultaneous monitoring of geomagnetic variations over the body and at a remote base station.

It is shown in this report that:

- Total field measurements are insufficient to determine the relative contributions of remanent and induced magnetisations to the anomaly, except in a qualitative fashion.
- Monitoring of all three field components at the on-anomaly and base stations, however, allows the components of k**A** to be determined, where k is the susceptibility of the source.
- Determination of the principal components (eigenvalues) and eigenvectors of k**A** provides information about the relative contributions of remanent and induced magnetisations to the anomaly.
- The following information can be obtained from the components of k**A** *without making any assumptions about source geometry or location*:
    - the Koenigsberger ratio (Q), which is the ratio of remanent magnetisation intensity to induced magnetisation intensity,
    - the direction of remanence,
    - the direction of total (remanent + induced) magnetisation.

This information can constrain magnetic modelling prior to drilling and remove a major source of ambiguity in magnetic interpretation. Applications include:

- Eliminating the well-known non-uniqueness of dip determination when the direction of magnetisation is unknown, reducing chances of drilling down dip.
- Determination of Q value, which constrains the geological nature of the source, by discriminating, for example between magnetite-bearing and pyrrhotite-bearing sources,
- Remote palaeomagnetism - remanence direction can discriminate between sources of different ages or magnetic mineralogy.

Thus the information provided by this method can substantially improve *geological* interpretation of magnetic anomalies and aid prioritisation of targets. A direct application to drill targetting of compact sources (for which the anomaly is dominated by the contribution of the dipole moment) is based upon the theoretical response of a sphere. The direction to the centre of a compact source can be determined directly from diagonalisation of the tensor. Repeating the procedure at another location within the magnetic anomaly can uniquely determine the location of the centre of the source, prior to drilling.

An alternative approach to remote determination of *in situ* magnetic properties and source location is to dispense with a remote base station and rely on simultaneous measurement of time-varying fields and their gradients at a single location within the anomaly. This procedure has the advantage of logistical simplicity and, most significantly, greatly ameliorates the requirements for accurate orientation of vector magnetometers, but requires highly sensitive gradiometers.

A number of mathematical relationships between component anomalies, which have application to processsing and interpretation of vector magnetic surveys, are derived in this report. It is shown that spatial derivatives of anomalous field components are not independent. Measurement of some gradient components can define the *local* values of other components uniquely. This provides a tighter constraint on undersampled component data than can be provided by a single component survey (e.g. a conventional total field survey) that is used to calculate component anomalies. The local spatial derivatives of anomlous components can be used to aid interpolation between survey lines, particularly if the line spacing is slightly too wide to adequately sample the field, and to calculate vertical gradients, which are useful for emphasising shallow features and defining geological boundaries.

11. References

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