one section of the data from San Juan, 7 yr for Hermanus and Kakioka. One first remark that can be made is that part of the solar-cycle effect is likely to be absorbed in the parabola, in which case Dst values cannot be used to derive solar-cycle changes (which is a goal mentioned by Sugiura & Poros or Mayaud). A second, maybe more important remark is that when a secular variation impulse occurs, this impulse cannot be removed by the single parabolic trend and will appear to some extent in the Dst data. The effect may fortunately be small in the case of the late 1960s impulse because two (and maybe even three) out of the four stations which are used to derive the Dst index are outside the zone which is affected by the impulse. This might not always be the case and demonstrates that one must be very careful in determining the true internal secular variation in order to derive an external index such as Dst.

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The response of palaeomagnetic data to Earth expansion

P. W. Schmidt and D. A. Clark *CSIRO Division of Mineral Physics, PO Box 136, North Ryde, NSW, Australia 2113*

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Summary. Attempts to estimate palaeo-radii of the Earth, using palaeomagnetic data have necessarily been based on simplistic Earth models. It has been asserted that real geological processes are too complex to enable us to approach the problem quantitatively, and such attempts yield invalid results. We examine this and argue that, to the contrary, it appears that errors introduced by allowing for more realistic behaviour of the continents, e.g. 'orange-peel effect' and crustal extension, are smaller by an order of magnitude than the response of palaeomagnetic data to simplified expansion models.

From a qualitative argument, it is shown that the observed Late Palaeozoic and Early Mesozoic palaeomagnetic data are not what should be expected from an expanded Earth. We conclude that it appears unlikely that the Earth has expanded significantly since the Early Mesozoic.

1 Introduction

Recently there has been renewed interest in the hypothesis that the Earth has expanded (Carey 1975, 1976; Owen 1976; McElhinny 1978; McElhinny, Taylor & Stevenson 1978). Although Egyed (1960) first realized that palaeomagnetic data should reflect any significant expansion, the most readily applicable method was developed by Ward (1963).

Ward's minimum dispersion method of palaeo-radius determination compares the scatter of palaeomagnetic pole positions calculated from coeval data as a function of ancient radius. The maximum value of k, the best estimate of the Fisherian precision parameter κ, is taken as corresponding to the most probable palaeo-radius for that time.

The problem of testing the statistical significance of the variation in k is similar to that of comparing dispersion before and after a folding correction has been applied. McElhinny (1964) has published 95 and 99 per cent confidence limits of the ratio k_2/k_1 for various samples sizes.

Applying this test to k values determined by Ward's method (Van Andel & Hospers 1968; McElhinny *et al.* 1978) for $R_a = R_p$ and $R_a = 0.8\,R_p$, it is apparent that in most cases the variation in k is not significant at the 95 per cent confidence level. Any one determination of palaeo-radius by this criterion, therefore, must be regarded as inconclusive. However, the large number of independent palaeo-radius determinations from all periods, now available, allows a mean estimated palaeo-radius to be calculated and standard statistical estimates of probable errors to be determined from the assumption that the mean values of R_a/R_p, being averages of many observations, are approximately normally distributed. Van Andel & Hospers (1968) and McElhinny *et al.* (1978) have published analyses of palaeo-radius estimations with confidence intervals such that all expansion hypotheses appear to be ruled out, at least since the Late Palaeozoic.

2 Crustal extension and the 'orange peel effect'

The question arises whether the null expansion result is not a consequence of the assumptions of Ward's model violating reality. It is clear, for example, that a continental block cannot simultaneously preserve total surface area, inter-site distances and inter-site angles whilst adjusting to the changing curvature of an expanding Earth. Carey (1976, p. 194) has elaborated this point and concluded that the obtained result is to be expected purely from the inadequacies of the model: 'As all the parameters assumed to be constant vary with the radius, at a rate which increases rapidly as the size of the triangle increases, the minimum scatter inevitably occurs with the least change from this base radius (R) in which $R_a/R = 1$. This indeed is what everyone has found who has used the method, and as everyone will find who uses the method in future. This predictable result has nothing to do with the former radius of the Earth'.

In a framework of a hierarchy of polygons, Carey (1976) envisages adjustments during expansion being accommodated between polygons on all levels, with the first order polygons increasing their dimensions predominantly through sea-floor spreading. He asserts (p. 194) that 'although intrinsically quite simple, and indeed predictable, the real-earth transformation is not simple mathematically'.

We believe that Carey's argument is cogent enough to necessitate some quantitative estimates of the effects of departures of Ward's model from a physically realistic adjustment of a continent to expansion. Two processes should serve to illustrate the continuum of adjustments which could take place: crustal extension by dyke injection or jointing leading to an increase in inter-site distance; and the so-called 'orange peel effect', where flattening of the continent as the radius increases and curvature decreases tends to produce radial tears and a consequent change in inter-site angular separation. Empirical limits can be placed on the magnitude of these effects from the observation that the continents have not significantly changed in size or shape since the Palaeozoic as evidenced by the success of reconstructions using present shape continents.

An idea of the effect of crustal extension on Ward's analysis can be obtained by considering the special case of two sites on a palaeo-meridian, one which expands along a radius (Ward's point of zero strain) and the other which retreats somewhat from the first by extension (Fig. 1). Denote the angle subtended by the sites at the centre of the Earth in the past and at present by θ_a and θ_p respectively, the palaeo- and present radius by R_a and R_p respectively, past and present site separations by S_a, S_p respectively, and define a crustal extension parameter δ by

$$\delta = 1 - \frac{1 - \theta_p/\theta_a}{1 - R_a/R_p} = \frac{1}{\theta_a}\,\frac{S_p - S_a}{R_p - R_a}. \tag{1}$$

$\delta = 0$ corresponds to constant inter-site distance (Ward's model) and $\delta = 1$ corresponds to pure radial dispersion of the sites, i.e. the continent has stretched such that the geocentric angle between sites is invariant, inter-site distance is proportional to the Earth's radius (an

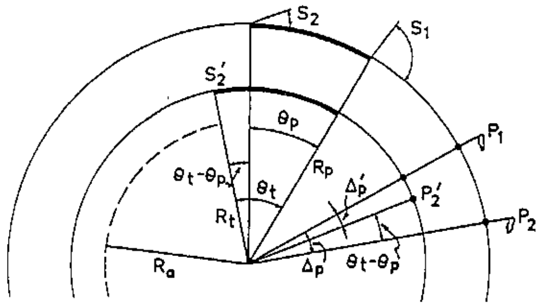

Figure 1. Cross-section showing geometrical relationships of test parameters (θ_t, R_t) to present parameters (θ_p, R_p). R_a is actual ancient radius. P_1 and P_2 are palaeomagnetic pole positions from palaeomagnetic inclinations at sites S_1 and S_2 respectively. P_2' is the palaeomagnetic pole from the recalculated site S_2' on an Earth of test radius R_t. S_1, for simplicity, is taken as the point of zero strain expanding radially.

impossibility if significant expansion has taken place, given the observed near constancy of continental areas).

The angular difference between the pole positions calculated from the two sites on the present Earth is given by (Fig. 1):

$$\Delta p = \theta_a - \theta_p = \theta_a(1-\delta)\,(1 - R_a/R_p). \tag{2}$$

Note that pure radial dispersion of sites implies undetectability of expansion since $\delta = 1$ implies $\Delta p = 0$ for all R_a/R_p.

Given the present angular separation of the two sites, Ward's method recalculates the site positions and palaeomagnetic poles for a test radius R_t. The new angular separation of sites θ_t and the corresponding difference in pole positions $\Delta p'$ are given by

$$\theta_t = \theta_p R_p/R_t \tag{3}$$

$$\Delta p' = \Delta p - (\theta_t - \theta_p) = \theta_a\,\{1 - [R_a + \delta(R_p - R_a)]/R_t\}. \tag{4}$$

$$\therefore \Delta p' = 0 \quad \text{for} \quad R_t = R_a + \delta(R_p - R_a) \tag{5}$$

Thus the effect of crustal extension ($\delta > 0$) is to bias palaeo-radius estimates upwards towards the present radius, but the effect is small for reasonable values of δ. If we consider $R_p/R_a = 1.25$, which is the order of expansion envisaged by Carey since the Permian, and 5 per cent increase in inter-site distance which corresponds to $\delta = 0.2$ (from equation (1)), then the estimated palaeo-radius from equation (5) is $R_t = 1.05R_a = 0.84R_p$, i.e. by disregarding the nominal 5 per cent increase in inter-site distance, the palaeo-radius is overestimated by 5 per cent.

The fact that values of R_t/R_p as low as 0.84 have not been found to be the most likely ratio using Ward's method strongly suggests that crustal extension alone cannot explain the null expansion result.

We now turn to the influence of the 'orange peel effect' on the dispersion of poles determined from sites on a continental block. Consider a continent in the form of a spherical cap when the rock units at the various sites were formed. A central point of the continent is regarded as a point of zero strain in Ward's model and the site localities are transformed to new coordinates with this central point as the pole. As the Earth expands the linear distances of the sites from the central point stay fixed (according to the model), whilst the parallels of latitude migrate radially away from this point. Ward assumes that the site longitudes measured from the central point remain constant. However, this is incompatible with the necessity of conserving the area covered by the continent, which tends to tear radially, thus changing the inter-site angles subtended at the continent centre. The extent of tearing can be estimated as follows.

The surface area of a spherical cap subtending angle θ_a at the centre of an Earth of radius R_a is

$$A = 2\pi R_a^2[1 - \cos(\theta_a/2)]. \tag{6}$$

As the Earth expands, the cap flattens to a spherical cap with a sector missing. Since the distance from the centre to the edge of the continent is unaltered, the angle subtended for radius R_p is $\theta_p = \theta_a R_a/R_p$. If f is the fraction of a full circle about the continent centre still covered by the continent after expansion, conservation of area gives

$$A = 2\pi R_a^2(1 - \cos\theta_a/2) = f\cdot 2\pi R_p^2(1 - \cos\theta_p/2) \tag{7}$$

$$\therefore f = (R_a^2/R_p^2)\,[(1 - \cos\theta_a/2)/(1 - \cos\theta_p/2)].$$

The total angle taken up by wedge-shaped tears is therefore $(1 - f) \times 360°$ and this could find its expression in one or two gaping rifts, which could correspond to Carey's sphenochasms. Such rifts would probably be recognizable and thus omitted from the analyses or corrected for; or else they would be distributed more or less evenly right around the continent leading to an insidious perturbation of site longitudes which would be difficult to detect and allow for.

However, the effect is small (except for very large expansion) as illustrated by taking $R_p/R_a = 1.25$ for a continent originally subtending 90° at the centre of the Earth. Then $f = 0.98$ and the maximum change in relative longitude of two sites is 7° if a single tear takes up the effect, or 3.5° between sites 180° apart in longitude if the effect is spread uniformly around the continent. In the worst case, this corresponds to pole positions being offset by $3.5° \sin(45°/1.25) = 2°$. Since the gross effect of the expansion on the calculated poles is given by $\Delta p = 18°$ for this case, the 'orange peel effect' is seen to be relatively unimportant. For smaller continents it is completely negligible, even given the greater precision of the data required to resolve palaeo-radii. However, it should be noted that Ward's method breaks down for sites greater than 90° away from the point of zero strain, as with a hemisphere sized continent, for example. None of the published applications of Ward's method are affected by this limitation, which is a consequence of the gross departure of the model from reality for very large inter-site distances.

3 Pole position invariance under expansion

A corollary to the undetectability of radial expansion of palaeomagnetic sites ($\Delta p = 0$ above) predicts that if the continents have dispersed radially then their pole paths should remain practically coincident, notwithstanding the second-order effects. Consider two rock units, of the same geological age, residing upon an expanding globe and dispersing radially. Their co-latitudes (p) are given by their magnetic inclinations (I) and the dipole equation (8)

$$\tan I = 2\cot p. \tag{8}$$

No matter what expansion occurs, their co-latitudes and pole positions remain invariant. This is depicted in Fig. 2 when, at some time after their formation, the rock units are rifted apart during expansion, but their *calculated* palaeomagnetic pole positions remain in agreement. This fact does not appear to be generally appreciated, but in qualitative fashion should be the observed consequence of the expansion hypothesis. The pole positions certainly should not be in better agreement after reconstructing the continents on the 'expanded' globe. However, it is an inescapable conclusion that the Late Palaeozoic and Early Mesozoic palaeomagnetic pole positions are in much better agreement after reconstructing Pangaea (Irving 1964; McElhinny 1973). Carey (1976, p. 3) indeed recognizes this in stating palaeomagnetic 'observations leave no doubt that the continents have separated'. The implication is that continental *drift* is necessary to explain the present distribution of the continents. Continental dispersion through Earth expansion could have played only a minor role.

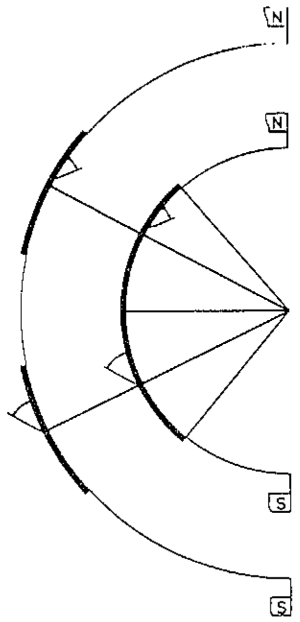

Figure 2. Invariance of palaeomagnetic poles from the magnetizations at radially dispersing sites. Although case shown is for sites along the same meridian, the relationship holds in general.

4 Conclusion

From both quantitative and qualitative arguments, significant Earth expansion since the Late Palaeozoic is unlikely. If expansion of the order required by Carey's model has occurred at some time during the Earth's history, then from palaeomagnetic data it would appear to have occurred before Late Palaeozoic times.

References

Carey, S. Warren, 1975. The expanding Earth: an essay review, *Earth Sci. Rev.*, 11, 105–143.
Carey, S. Warren, 1976. *The Expanding Earth*, 488 pages, Elsevier, Amsterdam.
Egyed, L., 1960. Some remarks on continental drift, *Geofis. pura Appl.*, 45, 115–116.
Irving, E., 1964. *Palaeomagnetism*, 399 pages, Wiley, New York.

McElhinny, M. W., 1964. Statistical significance of the Fold Test in Palaeomagnetism, *Geophys. J. R. astr. Soc.*, 8, 338–340.

McElhinny, M. W., 1973. *Palaeomagnetism and Plate Tectonics*, 358 pages, Cambridge University Press.

McElhinny, M. W., 1978. Limits to Earth expansion, *Bull. Aust. Soc. Explor. Geophys.*, 9(4), 149–152.

McElhinny, M. W., Taylor, S. R. & Stevenson, D. J., 1978. Limits to expansion of Earth, Moon, Mars and Mercury and to changes in the gravitational constant, *Nature*, 271, 316–321.

Owen, H. G., 1976. Continental displacement and expansion of the Earth during the Mesozoic and Cenozoic, *Phil. Trans. R. Soc. Lond. A*, 281 223–291.

Van Andel, S. I. & Hospers, J., 1968. A statistical analysis of ancient Earth radii calculated from palaeomagnetic data, *Tectonophysics*, 6, 491–497.

Ward, M. A., 1963. On detecting changes in the Earth's radius, *Geophys. J. R. astr. Soc.*, 8, 217–225.

Geophys. J. R. astr. Soc. (1980) 61, 101–113

Analysis of the modes of directional data with particular reference to palaeomagnetism

David R. Van Alstine *Division of Geological and Planetary Sciences,*
California Institute of Technology, Pasadena, California 91125, USA

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Summary. Advantages of using the mode in analysis of palaeomagnetic vectors are discussed, and a computer technique is described for contouring and precisely locating the modes of vector distributions that may be highly skewed. In contrast to conventional determinations of the mode, unit vectors from a given data set are treated not as discrete points, but as identical Fisherian probability density functions defined (at an angle θ from the unit vector) by: $p = \exp [sk(\cos \theta - 1)]$, where k is the estimate of the Fisherian concentration parameter, and s is an arbitrarily assigned 'smoothing parameter'. A grid, representing the cumulative probability distribution of the total sample of vectors, is contoured to provide a graphical display of the distribution around the most probable value, the mode. By repeatedly contouring the same sample of vectors with successively larger values of s, and by treating the mode as a vector with length given by the total probability value at the mode, 'progressive modal diagrams' can be constructed, to aid in determining the stable position of the mode of skewed distributions. In addition, a new statistic, 'β_{95}' is suggested as an error estimator for the mode. The statistic β_{95} is derived from the largest subset of the total sample that has a mean identical with the mode of the total sample; this statistic is defined as the Fisherian half-angle of the cone of 95 per cent confidence for the mean of this subset.

Introduction

Accurate determination of palaeomagnetic directions and poles is becoming increasingly important in regional and global tectonic syntheses and in stratigraphic correlation and age determination. Although the usual procedure for calculating these directions and poles has been to use statistical methods developed by Fisher (1953) and refined by Watson (1956a, 1956b, 1960, 1961, 1962, 1965, 1966) and Watson & Irving (1957), these methods assume a particular model of dispersion on a sphere roughly analogous to a two-dimensional, Gaussian (normal) distribution. Commonly, however, palaeomagnetists are confronted by distributions of vectors that are not symmetric about the mean. In these instances,