Advantages of Measuring the Magnetic Gradient Tensor

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Key words:

Aeromagnetic, survey, gradiometer, tensor, SQUID

Abstract

Measuring the magnetic gradient tensor will improve the interpretability of magnetic surveys, especially in areas where anomaly patterns are skewed by remanence or low magnetic latitudes. The benefits of total field gradiometry are well recognised, but the total magnetic intensity (TMI) is not a potential field and nor are its gradients. On the other hand, tensor components are true potential fields and possess desirable mathematical properties. The crucial difference between full tensor gradiometry and total field gradiometry is the production of more detailed and quantitatively interpretable maps and 3D models, rather than simple bump detection. Magnetics is still the cheapest and most widely used geophysical mapping tool in hard rock environments, with increasing importance and potential for further growth in hydrocarbon exploration.

Gradient tensor surveys will retain the benefits of vector surveys without the disadvantage of extreme orientation sensitivity. The tensor open up a wide range of new types of data processing techniques including application of invariants, directional filters, depth slicing, source moments and dipole location immune from sensor misorientation.

Superconducting interference devices (SQUIDs) are the sensors of choice for tensor gradiometry. They are vector sensors and are highly sensitive; they are small and consume little power. High-temperature SQUIDs (HTSs), which only require liquid nitrogen as opposed to liquid helium used for low-temperature SQUIDs, have intrinsic sensitivities ~100 fT (10^{-13} Tesla). It is also probable that in the future the required temperatures may be achieved without a cryogen. It has been estimated that gradiometer sensitivities of 0.01 nT/m can be achieved. This sensitivity is sufficient to detect anomalies over contacts between bodies with susceptibility contrasts as low as 60 x 10^{-5} SI at depths of over 100 m, and for contrasts of 600 x 10^{-5} SI at depths of over 1 km.

Tensor gradiometry will prove useful not only for aeromagnetic surveys, but also for environmental surveys, for defence applications such as submarine and unexploded ordnance detection and in down-hole magnetics. Any substantial improvement in this technology will have enormous benefits, in terms of new discoveries and lower exploration costs.

Introduction

Airborne magnetic surveys have improved dramatically over the past two decades with advances in both data acquisition and image processing techniques. Magnetic surveys form an integral part of exploration programs and are now routinely undertaken before geological mapping programs. These advances have been made despite treating the magnetic field as a scalar, wherein various processing procedures that assume a potential field are compromised. If the vector information could be retrieved, either by direct measurement or by mathematical manipulation, magnetic surveys could be improved even further. For instance, the total magnetic intensity (TMI) could be corrected so it represents a true potential field.

We discussed the calculation of vector components and lines-of-force from the TMI in a previous issue (Preview 70, October 1997, see also Schmidt and Clark, 1998) and implementing these techniques now forms part of a current AMIRA project (P602). Vector surveys, where the direct measurement of vector components has been attempted, have met with mixed success. The accuracy of direct measurement of the field vector is largely governed by orientation errors, which for airborne platforms are so large that the theoretical derivation of the components from the TMI is actually preferable. For this reason, and others listed below, it is desirable to measure the field gradient(s), rather than field vector.

Gradient measurements are relatively insensitive to orientation. This is because gradients arise largely from anomalous sources, and the background gradient is low. This contrasts with the field vector, which is dominated by the background field, i.e. that arising from the Earth's core. Gradient measurements are therefore most appropriate for airborne applications. Another advantage is they obviate the need for base stations and corrections for diurnal variations. They also greatly reduce the need for regional corrections, which are required by TMI surveys because of deeper crustal fields that are not of exploration interest, or the normal (quasi-) latitudinal intensity variation of the global field.

Gradient measurements also provide valuable additional information, compared to conventional total field measurements, when the field is undersampled. Undersampling is common perpendicular to flight lines in airborne surveys, is usual in ground surveys, and always pertains in down-hole surveys. Conditions under which calculation is preferable to measurement of vectors and gradient tensors have yet to be characterised by modelling and case studies. Synergistic interpretation of calculated vectors and measured gradients may allow significantly more information to be extracted from airborne surveys.

The advantages of magnetic gradients surveys are well known and include:

- Better resolution of shallow features and closely spaced sources
- Better definition of structural features
- Suppresses regional anomalies due to deep sources
- Subvertical contact mapper
- Anomalies tighter around compact sources
- Aids detection and delineation of pipe-like sources
- Constrains local strike direction*
- Determines on which side of line source lies*
- Common mode rejection of geomagnetic variations
- Relatively insensitive to rotation noise

- Constrains interpolation between flight lines* (important as all surveys are aliased to some extent across flight lines)
- IGRF corrections less important (usually unnecessary)
- Provides direct indication of Euler structural index when combined with measurements of field
- Higher resolution than conventional TMI surveys can be offset against survey height, allowing somewhat higher, therefore considerably safer, flying.

*not vertical TMI gradients

Total field gradiometry versus tensor gradiometry

Total field gradient surveys are common (Hood, 1981) and while they share many of the advantages of tensor gradients such as obviating or ameliorating the need for base stations and regional corrections, total field gradients are not vectors or true potential fields. Christensen and Rajagopalan (2000) suggest that the next breakthrough in magnetic exploration is likely to be the measurement of the gradient tensor.

To examine how the total field gradient and the gradient tensor are related, denote the regional geomagnetic field vector by F and the local field vector by F'. The anomalous field produced by subsurface sources is ΔB. Then

$$F' = F + \Delta B \tag{1}$$

The measured total field anomaly is given by:

$$\Delta B_m = |F'| - |F| \tag{2}$$

Traditionally, this is assumed to equal the projection of the anomalous field vector onto the regional field direction, $\Delta B_T = \Delta B \,.\, F/|F|$. ΔB_T has useful mathematical properties, because it is a potential field (it obeys Laplace's equation) and can be continued to other levels, if it is accurately known everywhere over one surface. In fact, the measured total field anomaly is equal to the ΔB_T only to first order in $\Delta B/F$. When anomalies are strong (thousands of nT), the difference between the two "total field" anomalies becomes significant. The maximum error due to equating the two quantities is:

$$\Delta B_m - \Delta B_T \approx (\Delta B)^2/2F \tag{3}$$

This implies a relative error of ~10% for a 10 000 nT anomaly in a 50 000 nT regional field.
Whereas ΔB_T obeys $\nabla^2(\Delta B_T) \equiv 0$ (Laplace's equation), the Laplacian of ΔB_m is given by:

$$\nabla^2(\Delta B_T) \approx [BXSIG^2 + BYSIG^2 + BZSIG^2 - ANSIG^2]/F \tag{4}$$

where $BXSIG$ is the analytic signal derived from ΔB_x (i.e. calculated using tensor components B_{xx}, B_{yx}, B_{zx}), $ANSIG$ is the analytic signal calculated from the total field gradients in the x, y and z directions etc. The RHS of the above expression is, in general, non-zero. For a body elongated parallel to y, $BYSIG^2 \approx 0$ and $BZSIG^2 \approx ANSIG^2$. Thus the RHS $\approx BXSIG^2/F > 0$. Because ΔB_m does not obey Laplace's equation exactly, it is not a potential field, and neither are its *derivatives* $(\partial \Delta B_m/\partial x,\ \partial \Delta B_m/\partial y,\ \partial \Delta B_m/\partial z)$. Specific expressions for the z-component and total field analytic signals are respectively:

$$BZSIG = \left(B_{zx}^2 + B_{zy}^2 + B_{zz}^2\right)^{1/2} \tag{5}$$

$$ANSIG = \left(\left(\frac{\partial B}{\partial x}\right)^2 + \left(\frac{\partial B}{\partial y}\right)^2 + \left(\frac{\partial B}{\partial z}\right)^2\right)^{1/2} \tag{6}$$

Specific advantages of magnetic tensor gradiometry and benefits that are specific to gradient tensor surveys include:

- Retains benefits of vector surveys, without disadvantage of extreme orientation sensitivity
- Tensor components are true potential fields, with desirable mathematical properties (important in areas with strong anomalies) - allows rigorous continuation, RTP, magnetisation mapping, etc
- Wide range of new types of processed data possible: invariants, directional filters, depth slicing, source moments and dipole location unaffected by sensor misorientation
- Each tensor component represents a directional filter, emphasising structures in particular orientations
- Combination of tensor components gives information on magnetisation directions
- Redundancy of tensor components gives inherent error correction and noise estimates
- Measurement of full tensor allows rotation of co-ordinate system, yielding transformed tensor components that emphasise specified structural orientations
- Allows direct determination of 3D analytic signal (defines source outlines; width/depth determinations - irrespective of remanent magnetisation)
- Measurement of tensor allows calculation of parameters with superior resolving power to conventional analytic signal
- Measurement of tensor allows calculation of parameters (e.g. invariants of the tensor) unaffected by aliasing across flight lines
- Superior Euler deconvolution solutions from measured tensor components with improved accuracy using true measured gradients along and across lines
- Tensor components are independent of skewing caused by geomagnetic field direction - ease of interpretability
- Defines direction to compact source directly from single station measurement
- Enables direct calculation of compact source magnetic moments
- Improved resolution of pipe-like bodies
- Improved resolution of sources subparallel to flight path
- Improved delineation of N-S elongated sources in low latitudes
- Spin-off applications to down-hole magnetics and remote determination of source magnetic properties *in situ*

Measurement of the gradient tensor

The most appropriate sensors for gradient measurements are Superconducting Quantum Interference Devices

(SQUIDs - see Foley *et al.*, 1999 and Foley and Leslie, 1998). SQUIDs detect minute changes of flux threading a superconducting loop. They are therefore variometers rather than magnetometers, but they are vector sensors since it is only changes perpendicular to the loop that are detected. So called high temperature SQUIDs, or HTSs, operate at liquid nitrogen temperatures (-197°C), overcoming the logistical problems of handing liquid helium. It is also probable that in the future the required temperatures may be achieved without a cryogen. Micro-miniature Joule-Thomson and low-power non-magnetic Stirling cryocoolers are being developed (Zimmerman, 1981).

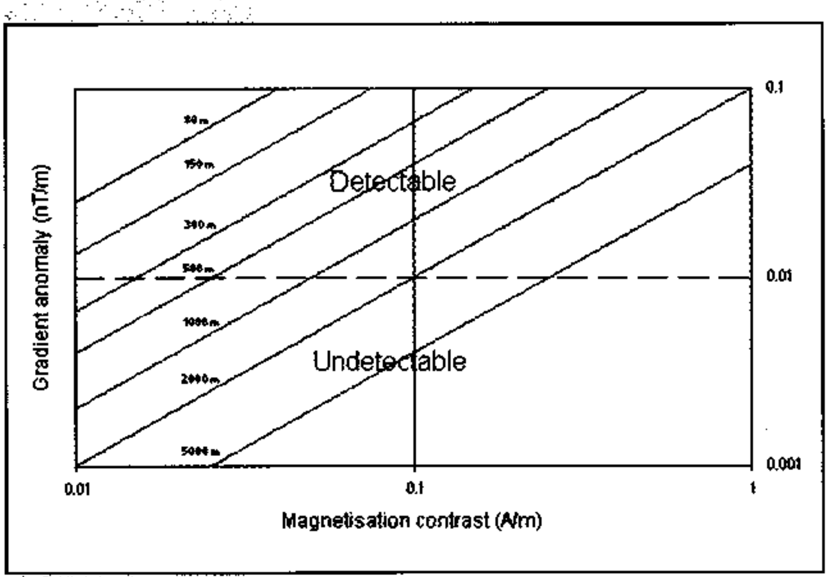

Fig. 1. Relationship between magnetization contrast across a contact and the vertical gradient anomaly of the total field. The graph is divided into detectable anomalies and undetectable anomalies for various depths from 80 m to 5000 m by the sensitivity level of 0.01 nT/m.

The sensitivity of SQUIDs is of the order of 100 fT (10^{-13} Tesla) and it has been estimated that gradiometer sensitivity should be better than 0.01 nT/m, on a baseline of 0.1 m. In Fig. 1 we have plotted the relationship between magnetization contrast across a vertical contact and the vertical gradient anomaly of the total field following Hood (1981). Although Hood's derivation is for total field anomalies over vertical contacts they are the same order of magnitude as gradient tensor components. In addition, the consideration of anomalies over vertical contacts is conservative because the Euler structural index, n, of a contact is only ~0.5, whereas for a thin dyke n = 1 and for a dipole n = 3. These higher structural indices translate into larger gradient anomalies.

For completeness however, Table 1 lists typical anomalies (assuming reduction to the pole for simplicity) of the gradient tensor component, B_{zz}. If we consider a vertical contact between two paramagnetic rock units such as a mafic and a felsic gneiss, which contain no magnetite or pyrrhotite, with a susceptibility contrast of ~60 × 10^{-5} SI, at 100 m the vertical field anomaly ΔB_z is 15 nT while ΔB_{zz} is -0.08 nT/m. This should be easily detected by a gradiometer with a sensitivity of 0.01 nT/m. If one rock unit contained ~0.2% magnetite the susceptibility contrast would be approximately 600 × 10^{-5} SI and detectable at depths of over 1 km.

SQUIDs are small (few cm) low power devices which may eventually find application down-hole or in drones. The very rapid sampling rate of SQUID sensors should allow unaliased detection of high frequency aircraft noise and efficient removal by filtering (total field magnetometers have much slower sampling, which is the cause of some compensation problems).

Source (Euler index)	ΔB_z	ΔB_{zz} (h = 100 m)	ΔB_{zz} (h = 500 m)
Sphere n = 3	100 nT	-3 nT/m	-0.6 nT/m
	10 nT	-0.3 nT/m	-0.06 nT/m
Pipe n = 2	100 nT	-2 nT/m	-0.4 nT/m
	10 nT	-0.2 nT/m	-0.04 nT/m
Dyke n = 1	100 nT	-1 nT/m	-0.2 nT/m
	10 nT	-0.1 nT/m	-0.02 nT/m
Vertical contact n = 0.5	100 nT	-0.5 nT/m	-0.1 nT/m
	10 nT	-0.05 nT/m	-0.01 nT/m

Table 1. Anomalies of the gradient tensor component, B_{zz}, assuming RTP.

Deployment of SQUIDs in aircraft and down-hole present different problems. Platform stability will need to be addressed in aircraft - GPS, tilt meters and other methods need to be assessed. Down-hole instruments will have to be slim (25 mm?) robust and reasonably affordable. SQUIDs potentially fulfil all these requirements.

In the real world the gradient tensor is a 3 x 3 second order tensor:

$$\begin{bmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{bmatrix} \tag{7}$$

In practice we only need to know five of the components. Because the divergence of the field is zero, i.e.

$$\nabla \cdot \mathbf{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \tag{8}$$

This means that the gradient tensor is traceless, and only two of the diagonal terms are required. In addition, in the absence of currents and any significant time variations in electrical fields, the curl of the field is also zero,

$$\nabla \times \mathbf{B} = \begin{bmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \mathbf{B}_x & \mathbf{B}_y & \mathbf{B}_z \end{bmatrix} = 0 \tag{9}$$

This implies that the gradient tensor is symmetric since the three orthogonal components of the curl are zero:

$$\frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0 \tag{10}$$

$$\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} = 0 \tag{11}$$

$$\frac{\partial B_y}{\partial z} - \frac{\partial B_z}{\partial y} = 0 \tag{12}$$

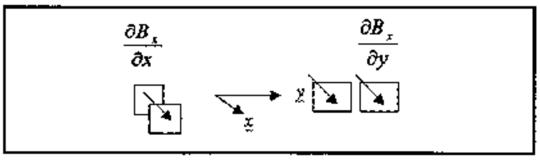

Fig. 2. Arrangement of SQUID sensors for detection of axial and transverse gradients.

Therefore only three off-diagonal terms are required. Fig. 2 depicts the arrangement of SQUID sensors for the detection of axial (diagonal) and transverse (off-diagonal) gradients. Detection of axial gradients requires two separate SQUID sensors but each transverse gradient can be detected using a single planar sensor, which greatly simplifies the total package. Thus seven SQUIDs are required in all to measure the magnetic gradient tensor.

Characteristics of tensor gradient components and derived quantities

In the following, the conventions used are:
+x = N; +y = E; +z = down.

- B_{xx} delineates E-W boundaries preferentially (symmetric for vertical magnetisation; antisymmetric for horizontal magnetisation)
- B_{yy} delineates N-S boundaries preferentially (symmetric for vertical magnetisation; antisymmetric for horizontal magnetisation)
- B_{xy} delineates body corners preferentially (anomaly signs depend on magnetisation direction)
- B_{zz} delineates steep boundaries preferentially (symmetric for vertical magnetisation; antisymmetric for horizontal magnetisation)
- B_{xz} delineates E-W boundaries preferentially (antisymmetric for vertical magnetisation; symmetric for N-S horizontal magnetisation)
- B_{yz} delineates N-S boundaries preferentially (antisymmetric for vertical magnetisation; symmetric for E-W horizontal magnetisation)
- The B_{ij} can be rotated into another co-ordinate system to resolve specific structural orientations
- Because B_{xz} and B_{yz} are acquired over a quasi-horizontal surface, they can be differentiated numerically to obtain $\partial B_{xz}/\partial x$ and $\partial B_{yz}/\partial y$. The second vertical derivative of ΔB_z, which has higher resolution than the first vertical derivative (B_{zz}), is easily calculated from these quantities:

$$B_{zzz} = \partial^2(\Delta B_z)/\partial z^2 = -\partial B_{xz}/\partial x - \partial B_{yz}/\partial y.$$

- The invariant I_1 outlines source boundaries and appears to have superior resolving power to the analytic signal. This is understandable, because of its faster fall-off

$$I_1 = B_{xx}B_{yy} + B_{yy}B_{zz} + B_{zz}B_{xx} - B_{xy}^2 - B_{yz}^2 - B_{zx}^2 \qquad (13)$$

- The invariant I_2 preferentially outlines shallower features of complex sources, because of its higher fall-off rate than I_1.

$$I_2 = DET\left[B_{ij}\right] - B_{xx}\left(B_{yy}B_{zz}\;\; B_{yz}^2\right) + B_{xy}\left(B_{yz}B_{xz} - B_{xy}B_{zz}\right) + B_{xz}\left(B_{xy}B_{yz} - B_{xz}B_{yy}\right\} \qquad (14)$$

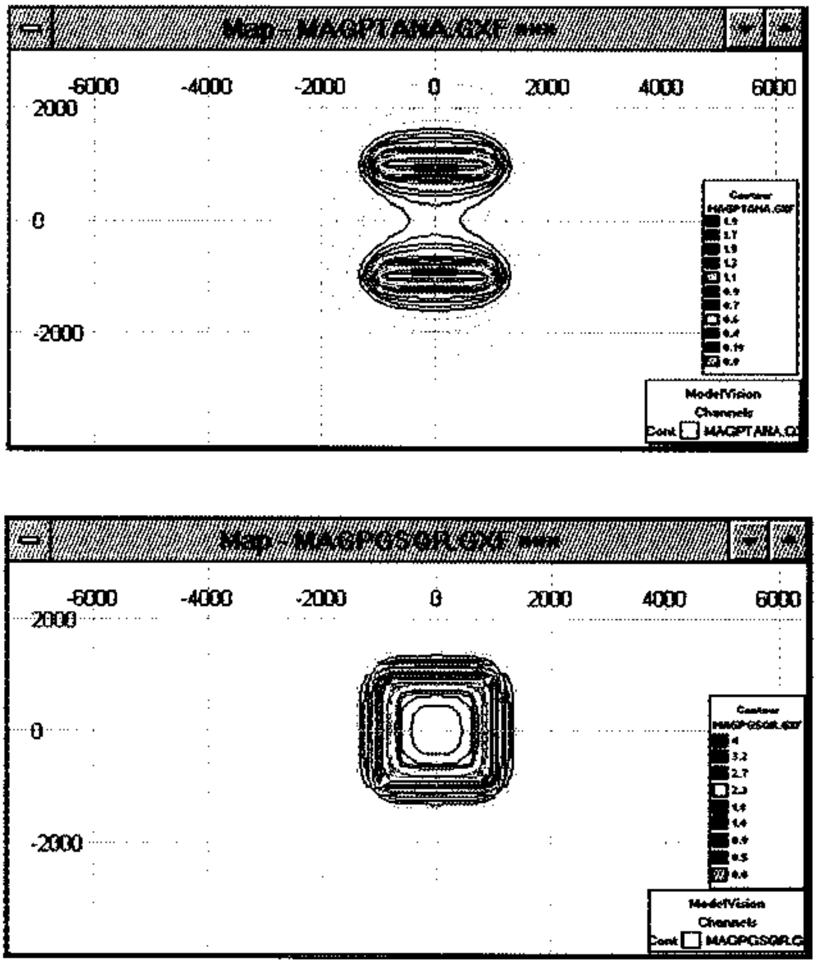

Fig. 3. Comparison of ANSIG and I_1 for a vertical prism model with vertical down (remanent) unit magnetisation at the (magnetic) equator.

The superior performance of the invariant I_1 is clearly shown in Fig. 3 which compares the *ANSIG* and I_1 for a vertical prism model with vertical down (remanent) unit magnetization at the (magnetic) equator. The ANSIG fails to detect the north-south sides of the prism, giving the appearance of two distinct bodies, while I_1 not only reveals these boundaries but also resolves them with greater clarity. Although the geometry chosen here is extreme it is emphasised that remanence should never be ignored and it is highly likely that somewhere in all surveys these or similar geometries exist.

Combined tensor/vector magnetometer packages

The tensor components along a short segment of a survey line or drill hole are sufficient to determine the location and magnetic moment of a compact (quasi-dipolar) source uniquely. There is insufficient information in $\nabla(\Delta B_m)$ to solve for these parameters. A tensor gradiometer sensor package could record field components (i.e. ΔB), as well as the gradients of these components, which would also allow direct determination of compact source location and moment.

Although small pods or veins of strongly magnetic material adjacent to a drill hole will produce intense gradients, the fall-off rate is very rapid. This implies:

- small magnetic bodies not in the immediate vicinity of the hole produce negligible effects

- pockets of magnetic material adjacent to the hole produce very localised spikes, easily distinguishable from the smoothly varying signature of large off-hole sources, particularly when combined with vector data.

A combined tensor/vector magnetometer package would allow the remote determination of in situ magnetic properties of sources from the surface or subsurface, using natural geomagnetic variations, without the alignment problems that afflict the differential vector magnetometer method (Clark, 1997; Clark et al., 1998).

Conclusions

There are many reasons why gradient tensor measurements will improve the interpretability of magnetic surveys, especially in areas where anomaly patterns are skewed by remanence or low magnetic latitudes. Gradient tensor surveys retain the benefits of vector surveys without the disadvantage of extreme orientation sensitivity. The tensor components are true potential fields with desirable mathematical properties. The tensor open up a wide range of new types of data processing techniques including invariants, directional filters, depth slicing, source moments and dipole location unaffected by sensor misorientation.

The crucial difference between full tensor gradiometry and total field gradiometry is the production of more detailed and quantitatively interpretable maps and 3D models, rather than simple bump detection. Magnetics is still the cheapest and most widely used geophysical mapping tool in hard rock environments, with increasing importance and potential for further growth in hydrocarbon exploration.

High-temperature SQUID sensors are well suited for tensor gradiometry. They are vector sensors and have high intrinsic sensitivities (~100 fT) and only require liquid nitrogen. Developments in cryocooler technology promise that even the cryogen may be dispensed with in the future. If gradiometer sensitivities of 0.01 nT/m can be achieved then anomalies over vertical contacts (structural index ~0.5) between bodies with a susceptibility contrast as low as 60 x 10^{-5} SI can be detected at depths of over 100 m. 60 x 10^{-5} SI is a weak susceptibility contrast. Obviously anomalies over bodies with greater susceptibilty contrasts and/or higher structural indices can be detected at greater depths.

Tensor gradiometry will prove useful for aeromagnetic surveys with wide line spacings (e.g. over sedimentary basins), environmental surveys, defence applications such as submarine and unexploded ordnance detection and down-hole magnetics.

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