Correction of Electric and Magnetic Fields and Gradients Measured Within and Around an Insulating Sensor Capsule in Seawater

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Abstract- **The presence of a highly conductive medium around a measurement capsule influences electromagnetic measurements made in the ocean and fundamentally alters the structure of the magnetic gradient tensor. Additional effects arise if seawater is flowing past the sensor package. This paper presents a quantitative analysis of these effects and describes the corrections that need to be applied to obtain accurate absolute measurements of electromagnetic fields and gradients in the ocean. For example, for a small spherical cavity within a 1D horizontal quasistatic electric current distribution, the electric field within the cavity is parallel to the unperturbed applied field and larger by 50%, and the magnetic field at the centre of the cavity is equal to the unperturbed magnetic field that existed at the same point in the conductive medium, prior to insertion of the measurement capsule. The symmetric magnetic gradient tensor within the cavity is uniform. If the unperturbed electric current is parallel to the x axis, the only non-zero components of the magnetic gradient tensor within the cavity are $B_{yz} = B_{zy}$. These components are each equal to half the value of $\partial B_y/\partial z$ that is produced by the unperturbed current flow in the conductive medium. The external perturbation of the electric field around the cavity has the configuration of a dipole field and the external magnetic field due to the cavity is that of an elementary current dipole. An ellipsoidal cavity has an anisotropic response, except in the degenerate case where all axes are equal and the cavity is spherical. Unless the applied field lies along a principal axis of the ellipsoid, the internal field is not parallel to the applied field, but is deflected away from the major axis and towards the minor axis. An applied electric field is amplified within the cavity. For a disk-like cavity the amplification of the applied electric field normal to the disk can be very large within the cavity. The anomalous magnetic field within the ellipsoidal cavity due to electric current flow around the cavity is nonuniform, but has a uniform gradient. At the center of the ellipsoidal cavity the magnetic field is equal to the field that existed at that point before insertion of the cavity. The *resultant* internal magnetic gradient tensor is symmetric and traceless, as required. Seawater motion past a sensor package produces easily detectable effects that can represent an important source of electromagnetic noise. In the vicinity of a measurement capsule, water velocities of the order of 1 ms^{-1} produce perturbations of ~45 µVm^{-1} in electric field, ~75 pT in magnetic field, and produce magnetic gradients of ~150 pT/m.**

INTRODUCTION

The CSIRO Division of Materials Science and Engineering is developing sensitive magnetic tensor gradiometers, based on high-temperature SQUID technology, for deployment in the marine environment. The applications include gradient measurements as an adjunct to electric and magnetic measurements in marine CSEM surveys, unexploded ordnance detection in shallow water, and exploration for seafloor mineralization.

Electric (**E**) and magnetic (**B**) fields within a conductive medium are perturbed by the measurement process. In particular, sensors located within or around an insulating measurement capsule measure fields that are modified by the diversion of conduction currents around the capsule. For quasistatic field measurements in air or free space, where conduction and convection currents are absent and displacement currents are negligible, the magnetic gradient tensor, which has elements $B_{ij} = \partial B_j/\partial x_i$ (i,j = 1,2,3), is symmetric, as well as traceless. In this case the tensor only has five independent components (e.g. B_{11}, B_{22}, B_{12}, B_{13}, B_{23}). On the other hand, in the presence of conduction currents the curl of **B** is non-zero and the gradient tensor is asymmetric, with eight independent components.

This raises the question of what is actually measured by magnetometers and gradiometers immersed in the electrically conductive ocean. In particular, how does the signal measured within a sealed capsule (within which the gradient tensor is symmetric) relate to the field components and the asymmetric gradient tensor that existed in the surrounding medium prior to insertion of the measurement package? The answer to this question depends on the configuration of the sensors, in particular whether conduction currents flow between individual sensors or the entire sensor package is contained within an insulating capsule. In the quasistatic limit, which applies to measurements made for magnetotelluric and typical marine controlled source EM surveys, and to oceanographic applications, the effects depend only on geometry and are independent of frequency.

This paper presents theoretical relationships between measured electric and magnetic fields and gradients and the corresponding quantities that would exist in the unperturbed medium, for a variety of geometries, including ellipsoidal measurement capsules. The effects of water flow around the capsule are also considered.

EFFECT OF AN ELLIPSOIDAL CAVITY WITHIN A CONDUCTIVE MEDIUM

Consider an infinite ohmic medium of conductivity σ subject to a uniform applied electric field $\mathbf{E}_0$. Insertion of an insulating measurement capsule into the initially uniform electric current distribution, of density $\mathbf{j}_0 = \sigma\mathbf{E}_0$, distorts the current flow and the associated electric field. Ellipsoidal cavities can be used to model a wide variety of capsule shapes, whilst conveniently allowing analytic solutions. Let us represent the capsule by a triaxial ellipsoidal cavity, centered at the origin, with semiaxes $a > b > c$ along x_1, x_2, x_3 respectively. The ellipsoidal coordinates ξ, η, ζ are the roots, in descending order, of the cubic equation in λ

$$\frac{x_1^2}{a^2+\lambda}+\frac{x_2^2}{b^2+\lambda}+\frac{x_3^2}{c^2+\lambda}=1, \tag{1}$$

where $\xi > -c^2$, $-c^2 > \eta > -b^2$, $-b^2 > \zeta > -a^2$ ([1], [2]). The boundary of the ellipsoidal cavity is defined by $\xi = 0$. In terms of Cartesian coordinates, the potential V_0 associated with a uniform applied electric field $\mathbf{E}_0$ is

$$V_0 = -\mathbf{E}_0.\mathbf{r} = -(\mathbf{E}_0)_1 x_1 - (\mathbf{E}_0)_2 x_2 - (\mathbf{E}_0)_3 x_3. \tag{2}$$

In terms of ξ, η, ζ, the applied potential is given by

$$\begin{aligned} V_0 = &-(\mathbf{E}_0)_1\sqrt{\frac{(\xi+a^2)(\eta+a^2)(\zeta+a^2)}{(a^2-b^2)(a^2-c^2)}} \\ &-(\mathbf{E}_0)_2\sqrt{\frac{(\xi+b^2)(\eta+b^2)(\zeta+b^2)}{(c^2-b^2)(a^2-c^2)}} \\ &-(\mathbf{E}_0)_3\sqrt{\frac{(\xi+c^2)(\eta+c^2)(\zeta+c^2)}{(b^2-c^2)(a^2-c^2)}}. \end{aligned} \tag{3}$$

The electric potential V obeys Laplace's equation everywhere, is continuous at the cavity boundary, asymptotically approaches V_0 at large distances ($\xi \sim r^2 >> a^2$), and is subject to the boundary condition $\partial V/\partial n \to 0$ as $\xi \to 0^+$, since the external electric field is proportional to the electric current within the ohmic medium and no current crosses the cavity boundary. Solving the boundary value problem in ellipsoidal coordinates gives

$$V(\xi\le 0) = -\frac{(\mathbf{E}_0)_1}{1-D_1}x_1 - \frac{(\mathbf{E}_0)_2}{1-D_2}x_2 - \frac{(\mathbf{E}_0)_{3}}{1-D_3}x_3, \tag{4}$$

for the internal potential, and

$$V(\xi\ge 0) = V_1 + V_2 + V_3 \tag{5}$$

for the external potential, where

$$\begin{aligned} V_1(\xi\ge 0) &= -(\mathbf{E}_0)_1 x_1\left[1+\frac{abc/2}{1-D_1}\int_\xi^\infty \frac{ds}{(s+a^2)R_s}\right] \\ &= -(\mathbf{E}_0)_1 x_1\left[1+\frac{abc/2}{1-D_1}A(\xi)\right], \end{aligned} \tag{6}$$

$$\begin{aligned} V_2(\xi\ge 0) &= -(\mathbf{E}_0)_2 x_2\left[1+\frac{abc/2}{1-D_2}\int_\xi^\infty \frac{ds}{(s+b^2)R_s}\right] \\ &= -(\mathbf{E}_0)_2 x_2\left[1+\frac{abc/2}{1-D_2}B(\xi)\right], \end{aligned} \tag{7}$$

$$\begin{aligned} V_3(\xi\ge 0)_1 &= -(\mathbf{E}_0)_3 x_3\left[1+\frac{abc/2}{1-D_3}\int_\xi^\infty \frac{ds}{(s+c^2)R_s}\right] \\ &= -(\mathbf{E}_0)_3 x_3\left[1+\frac{abc/2}{1-D_3}C(\xi)\right]. \end{aligned} \tag{8}$$

In (4)-(6) $R_s = \sqrt{(s+a^2)(s+b^2)(s+c^2)}$. The D_i (i = 1,2,3) are demagnetizing (or depolarizing) factors of the ellipsoid along its major, intermediate and minor axes respectively. Explicit expressions in terms of standard elliptic integrals for $D_1 = abcA(0)/2$, $D_2 = abcB(0)/2$, $D_3 = abcC(0)/2$, and for the functions $A(\xi)$, $B(\xi)$, $C(\xi)$ are given in [3]. If two or more axes are equal, or if an axis becomes infinite, the elliptic integrals reduce to elementary functions.

Equation (2) implies a uniform internal field given by:

$$\begin{aligned} \mathbf{E}(\xi\le 0) = -\nabla V = &\left(\frac{1}{1-D_1}\right)(\mathbf{E}_0)_1\hat{\mathbf{x}}_1 \\ &+\left(\frac{1}{1-D_2}\right)(\mathbf{E}_0)_2\hat{\mathbf{x}}_2+\left(\frac{1}{1-D_3}\right)(\mathbf{E}_0)_3\hat{\mathbf{x}}_3. \end{aligned} \tag{9}$$

The demagnetizing factors sum to unity. Since $D_1 \le D_2 \le D_3$, an ellipsoidal cavity has an anisotropic response, except in the degenerate case where all axes are equal and the cavity is spherical. In that case $D_1 = D_2 = D_3 = 1/3$. Unless the applied field lies along a principal axis of the ellipsoid, the internal field is not parallel to the applied field, but is deflected away from the major axis and towards the minor axis. It is obvious from (9) that an applied electric field is amplified within the cavity. For a disk-like cavity $D_3 \to 1$ as $c/a \to 0$, so the amplification of the applied electric field normal to the disc can be very large within the cavity.

The corresponding external field components are obtained from (5)-(8) by differentiation:

$$E_1(\xi \geq 0) = (\mathbf{E}_0)_1 \left\{ \left[1 + \frac{abc/2}{1-D_1} A(\xi) \right] + \frac{abcA'(\xi)x_1}{2(1-D_1)} \frac{\partial \xi}{\partial x_1} \right\}$$
$$+ \left[(\mathbf{E}_0)_2 \frac{abcB'(\xi)x_2}{2(1-D_2)} + (\mathbf{E}_0)_3 \frac{abcC'(\xi)x_3}{2(1-D_3)} \right] \frac{\partial \xi}{\partial x_1}, \quad (10)$$

$$E_2(\xi \geq 0) = (\mathbf{E}_0)_2 \left\{ \left[1 + \frac{abc/2}{1-D_2} B(\xi) \right] + \frac{abcB'(\xi)x_2}{2(1-D_2)} \frac{\partial \xi}{\partial x_2} \right\}$$
$$+ \left[(\mathbf{E}_0)_1 \frac{abcA'(\xi)x_1}{2(1-D_1)} + (\mathbf{E}_0)_3 \frac{abcC'(\xi)x_3}{2(1-D_3)} \right] \frac{\partial \xi}{\partial x_2}, \quad (11)$$

$$E_3(\xi \geq 0) = (\mathbf{E}_0)_3 \left\{ \left[1 + \frac{abc/2}{1-D_3} C(\xi) \right] + \frac{abcC'(\xi)x_3}{2(1-D_1)} \frac{\partial \xi}{\partial x_3} \right\}$$
$$+ \left[(\mathbf{E}_0)_2 \frac{abcB'(\xi)x_2}{2(1-D_2)} + (\mathbf{E}_0)_1 \frac{abcA'(\xi)x_1}{2(1-D_1)} \right] \frac{\partial \xi}{\partial x_3}. \quad (12)$$

Reference [3] gives explicit expressions for the demagnetizing factors, the functions $A(\xi)$, $B(\xi)$, $C(\xi)$ and their derivatives with respect to ξ, and the derivatives $\partial\xi/\partial x_i$. For a spherical cavity D_i = 1/3, so (9) implies that the internal electric field is parallel to the applied field, but amplified by a factor of 1.5. The *external* field around a spherical cavity is identical to that of an elementary current dipole located at the center of the cavity, immersed in an infinite homogeneous conductive medium, in the limit as frequency goes to zero [4]. The moment **p** of this equivalent current dipole is

$$\mathbf{p} = I\Delta\mathbf{x} = -2\pi a^3 \mathbf{j}_0 = -2\pi a^3 \sigma \mathbf{E}_0. \quad (13)$$

where Δx (<< a) is the length and I is the current carried by the dipole.

Fig.1 shows the variation of the local electric field around a spherical cavity. Also shown is the average electric field over a finite baseline, which is what is measured by a standard marine electrometer that estimates the electric field from the potential difference between two electrodes, symmetrically disposed about an electronics capsule, divided by the electrode separation. For a long cylindrical cavity, oriented perpendicular to the applied field, the internal field is also parallel to the applied field, but in this case is approximately doubled in strength, because $D_2 = D_3 = 1/2$ for an infinite cylinder.

For a spherical cavity, the perturbation of the electric current corresponds to a ball of impressed current $\mathbf{j}' = -\mathbf{j}_0$, with a return flow that conforms to a dipole field. In this case the anomalous magnetic field $\mathbf{B}'$ due to the perturbation of the electric current distribution can be calculated by applying Ampere's circuital law to circular contours centered on an axis passing through the sphere, parallel to the applied electric field. In terms of spherical polar coordinates r, θ, φ with the polar axis along x_1, parallel to the applied electric field, the anomalous internal magnetic field is

$$B'_\varphi(r \leq a) = \frac{-\mu_0 j_0 r \sin\theta}{2} = \frac{-\mu_0 j_0 \sqrt{x_2^2 + x_3^2}}{2}. \quad (14)$$

The nonzero Cartesian components are

$$B'_2(r \leq a) = \frac{\mu_0 j_0 x_3}{2}, B'_3(r \leq a) = -\frac{\mu_0 j_0 x_2}{2}, \quad (15)$$

and the nonzero anomalous gradient tensor elements are

$$B'_{23}(r \leq a) = -\frac{\mu_0 j_0}{2}, B'_{32}(r \leq a) = \frac{\mu_0 j_0}{2}. \quad (16)$$

The anomalous magnetic field $\mathbf{B}'$ and corresponding gradient tensor $\mathbf{G}'$ within a spherical cavity can also be written in coordinate-free form as

$$\mathbf{B}'(r \leq a) = \frac{\mu_0(\mathbf{r} \times \mathbf{j}_0)}{2}, \quad (17)$$

and

$$\mathbf{G}'(r \leq a) = \frac{\mu_0 \mathbf{I} \times \mathbf{j}_0}{2}, \quad (18)$$

where $\mathbf{I} = \hat{\mathbf{x}}_1\hat{\mathbf{x}}_1 + \hat{\mathbf{x}}_2\hat{\mathbf{x}}_2 + \hat{\mathbf{x}}_3\hat{\mathbf{x}}_3$ is the identity dyadic or idemfactor.

In vector form the anomalous external magnetic field is

$$\mathbf{B}'(r \geq a) = \frac{\mu_0 a^3 (\mathbf{r} \times \mathbf{j}_0)}{2r^3}, \quad (19)$$

and the gradient tensor is

$$\mathbf{G}'(r > a) = \frac{\mu_0 a^3}{2} \left[\frac{1}{r^3} \mathbf{I} \times \mathbf{j}_0 - \frac{3\mathbf{r}}{r^5} (\mathbf{r} \times \mathbf{j}_0) \right]. \quad (20)$$

The magnetic field configuration given by (19) is identical to that of an infinitesimal current element, given by the Biot-Savart law, which is applicable in this case because the contributions of the return currents in the medium integrate to zero around the return paths [5].

Ampere's circuital law cannot be used in the same way for a nonspherical cavity, due to the lower symmetry of the problem. Consider initially an ellipsoidal cavity with major axis aligned with the applied electric field. The volume of the cavity τ = $4\pi abc/3$. The effect of the ellipsoidal cavity can be regarded as the superposition of an ellipsoidal distribution of impressed current $\mathbf{j}' = -\mathbf{j}_0$, together with its ohmic return currents in the surrounding medium, and the uniform applied current flow $\mathbf{j}_0$. The anomalous magnetic field $\mathbf{B}'$ arising from the perturbation $\mathbf{j}'$ of the current distribution is continuous at the cavity surface and satisfies $\nabla\times \mathbf{B}' = \mu_0\mathbf{j}'$ and $\nabla . \mathbf{B}' = 0$ everywhere inside and outside the ellipsoid. Outside the ellipsoid $\mathbf{j}'$ is equal to $\sigma(\mathbf{E}-\mathbf{E}_0)$, where $\mathbf{E}$ is given by (10)-(12). The components of $\mathbf{B}'$ that satisfy these conditions are

$$B_1' = 0, \quad \forall \xi \tag{21}$$

$$\begin{aligned} B_2'(\xi < 0) &= \frac{\mu_0 j_1 abc C(0)}{2(1-D_1)} x_3 \\ &= \frac{\mu_0 j_1 abc C(0)}{2(1-D_1)} \sqrt{\frac{(\xi + c^2)(\eta + c^2)(\zeta + c^2)}{(a^2 - c^2)(b^2 - c^2)}}, \end{aligned} \tag{22}$$

$$\begin{aligned} B_3'(\xi < 0) &= -\frac{\mu_0 j_1 abc B(0)}{2(1-D_1)} x_2 \\ &= -\frac{\mu_0 j_1 abc B(0)}{2(1-D_1)} \sqrt{\frac{(\xi + b^2)(\eta + b^2)(\zeta + b^2)}{(a^2 - c^2)(b^2 - c^2)}}, \end{aligned} \tag{23}$$

$$\begin{aligned} B_2'(\xi \geq 0) &= \frac{\mu_0 j_1 abc C(\xi)}{2(1-D_1)} x_3 \\ &= \frac{\mu_0 j_1 abc C(\xi)}{2(1-D_1)} \sqrt{\frac{(\xi + c^2)(\eta + c^2)(\zeta + c^2)}{(a^2 - c^2)(b^2 - c^2)}}, \end{aligned} \tag{24}$$

$$\begin{aligned} B_3'(\xi \geq 0) &= -\frac{\mu_0 j_1 abc B(\xi)}{2(1-D_1)} x_2 \\ &= -\frac{\mu_0 j_1 abc B(\xi)}{2(1-D_1)} \sqrt{\frac{(\xi + b^2)(\eta + b^2)(\zeta + b^2)}{(a^2 - c^2)(b^2 - c^2)}}. \end{aligned} \tag{25}$$

The anomalous field given by (21)-(25) is solenoidal, has the correct curl everywhere and is continuous across the cavity boundary (ξ = 0), as required. It is therefore the unique solution to the boundary value problem. As confirmation, the anomalous magnetic field, given by (24)-(25), conforms to the field of a current dipole at large distances. Noting that for $r >> a$, $\xi \to r^2$ and $A(\xi)$, $B(\xi)$, $C(\xi) \to 2/3r^3$, asymptotic expressions for the field components are

$$B_2'(\xi >> a^2) \sim \frac{\mu_0 j_1 abc}{3(1-D_1)} \frac{x_3}{r^3} = \frac{\mu_0 j_1 \tau}{4\pi(1-D_1)} \frac{x_3}{r^3}, \tag{26}$$

$$B_3'(\xi >> a^2) \sim -\frac{\mu_0 j_1 abc}{3(1-D_1)} \frac{x_2}{r^3} = \frac{-\mu_0 j_1 \tau}{4\pi(1-D_1)} \frac{x_2}{r^3}. \tag{27}$$

It can be seen by comparison with (19) that the expressions on the RHS of (26) and (27) are consistent with the corresponding expressions for a spherical cavity of equal $\tau/(1-D_1)$, where $D_1 = 1/3$ for a sphere.

So far only a specific orientation of the ellipsoid has been considered. In general the ellipsoid may have arbitrary orientation with respect to the current flow. If the components of the unperturbed current flow with respect to the ellipsoid axes are j_1, j_2, j_3 then the resultant anomalous field represents the superposition of the effects that would be produced by currents along each of the ellipsoid axes. The result, obtained analogously to (21)-(25), is

$$\begin{aligned} B_1'(\xi < 0) &= \frac{\mu_0 j_3 abc B(0)}{2(1-D_3)} x_2 - \frac{\mu_0 j_2 abc C(0)}{2(1-D_2)} x_3 \\ &= \mu_0 \left(\frac{j_3 D_2 x_2}{(1-D_3)} - \frac{j_2 D_3 x_3}{(1-D_2)} \right), \end{aligned} \tag{28}$$

$$\begin{aligned} B_2'(\xi < 0) &= \frac{\mu_0 j_1 abc C(0)}{2(1-D_1)} x_3 - \frac{\mu_0 j_3 abc A(0)}{2(1-D_3)} x_1 \\ &= \mu_0 \left(\frac{j_1 D_3 x_3}{(1-D_1)} - \frac{j_3 D_1 x_1}{(1-D_3)} \right), \end{aligned} \tag{29}$$

$$\begin{aligned} B_3'(\xi < 0) &= \frac{\mu_0 j_2 abc A(0)}{2(1-D_2)} x_1 - \frac{\mu_0 j_1 abc B(0)}{2(1-D_1)} x_2 \\ &= \mu_0 \left(\frac{j_2 D_1 x_1}{(1-D_2)} - \frac{j_1 D_2 x_2}{(1-D_1)} \right), \end{aligned} \tag{30}$$

$$\begin{aligned} B_1'(\xi > 0) &= \frac{\mu_0 j_3 abc B(\xi)}{2(1-D_3)} x_2 - \frac{\mu_0 j_2 abc C(\xi)}{2(1-D_2)} x_3 \\ &= \frac{3\mu_0 \tau}{8\pi} \left(\frac{B(\xi) j_3 x_2}{(1-D_3)} - \frac{C(\xi) j_2 x_3}{(1-D_2)} \right), \end{aligned} \tag{31}$$

$$\begin{aligned} B_2'(\xi > 0) &= \frac{\mu_0 j_1 abc C(\xi)}{2(1-D_1)} x_3 - \frac{\mu_0 j_3 abc A(\xi)}{2(1-D_3)} x_1 \\ &= \frac{3\mu_0 \tau}{8\pi} \left(\frac{C(\xi) j_1 x_3}{(1-D_1)} - \frac{A(\xi) j_3 x_1}{(1-D_3)} \right), \end{aligned} \tag{32}$$

$$\begin{aligned} B_3'(\xi > 0) &= \frac{\mu_0 j_2 abc A(\xi)}{2(1-D_2)} x_1 - \frac{\mu_0 j_1 abc B(\xi)}{2(1-D_1)} x_2 \\ &= \frac{3\mu_0 \tau}{8\pi} \left(\frac{A(\xi) j_2 x_1}{(1-D_2)} - \frac{B(\xi) j_1 x_2}{(1-D_1)} \right). \end{aligned} \tag{33}$$

The anomalous magnetic field within the ellipsoidal cavity is nonuniform, but has a uniform gradient. At the center of the ellipsoidal cavity ($x_1 = x_2 = x_3 = 0$) the magnetic field is equal to the field that existed at that point before insertion of the cavity. The *resultant* internal magnetic gradient tensor is symmetric and traceless, as required. Its components are given by

$$\begin{aligned} G_{ii} &= G_{ii}^0, (i = 1,2,3) \\ G_{12} &= G_{21} = G_{21}^0 + \mu_0 D_2 j_3 / (1-D_3) \\ G_{13} &= G_{31} = G_{13}^0 + \mu_0 D_1 j_2 / (1-D_2) \\ G_{23} &= G_{32} = G_{32}^0 + \mu_0 D_3 j_1 / (1-D_1) \end{aligned} \tag{34}$$

where the unperturbed gradient tensor is $\mathbf{G}_0 = [\, G_{ij}^0 \,]$, which depends on the configuration of the unperturbed current flow, on geological anomalies and on other magnetic sources. As an example, Fig.2 shows magnetic gradient tensor components within and around a spherical cavity placed within a 1D current distribution in the ocean. These results allow measurements by sensors that have been calibrated in air to be corrected for the effect of a conductive medium. For a capsule adjacent to a conductivity interface, such as the sea surface or seafloor, the interaction effects can be approximately evaluated using the method of images.

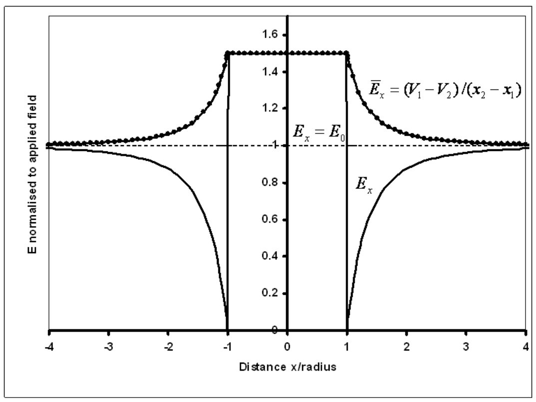

Fig.1. Normalized electric field (E) profile, parallel to the uniform applied field, passing through the centre of a spherical cavity. Solid line indicates the local field; the horizontal dashed line represents the unperturbed applied field; the line with dots shows the average field over a baseline defined by symmetrically placed potential electrodes.

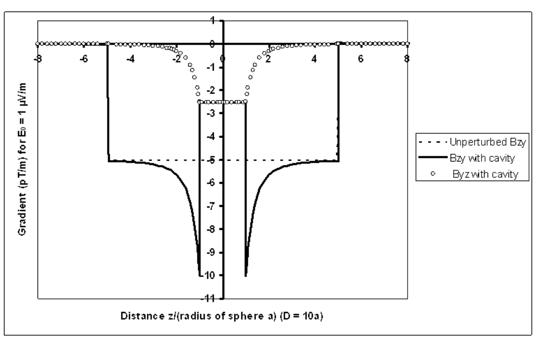

Fig.2. Magnetic gradient tensor elements along a vertical profile through the centre of a spherical cavity within a horizontal current flow distribution of limited depth extent. The assumed conductivity of the seawater is 4 $\mathrm{Sm^{-1}}$. Magnetic gradients are normalized to an applied electric field of 1 μV/m. The current flow is confined to an infinite horizontal slab of thickness D equal to five times the diameter of the cavity. Within the current flow, but beyond the influence of the cavity, the only nonzero gradient tensor component is B_{zy} (solid line). The dashed horizontal line is the unperturbed value of B_{zy} within the 1D current distribution. The dots indicate B_{yz}. Within the cavity $B_{zy} = B_{yz}$. The influence of the cavity is significant out to $r \approx 3a$.

ELECTRIC AND MAGNETIC EFFECTS OF WATER FLOW

So far it has been assumed that the conductive medium is stationary. If the seawater is moving with respect to the sensor with velocity **v**, however, Maxwell's equations must be modified to take the Lorentz field into account. Scaling arguments show that the conduction current totally dominates other sources of magnetic field in the ocean and that the contribution of electromagnetic induction to the electric field is minor at low frequencies and for quasistatic water flow [6]. We consider steady state water flow and static fields and neglect transport of free charges, which can be shown to be effectively absent within the seawater. Maxwell's equations for moving media are then [7]

$$\nabla.(\varepsilon_0 \mathbf{E} + \mathbf{P}) = 0, \tag{35}$$

$$\nabla \times \mathbf{E} = \mathbf{0}, \tag{36}$$

$$\nabla.\mathbf{B} = 0, \tag{37}$$

$$\nabla \times \mathbf{B} = \mu_0 [\mathbf{j} + \nabla \times (\mathbf{P} \times \mathbf{v})], \tag{38}$$

with constitutive equations

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{F}), \tag{39}$$

$$\mathbf{P} = (\varepsilon - \varepsilon_0)(\mathbf{E} + \mathbf{v} \times \mathbf{F}), \tag{40}$$

where $\mathbf{E} = -\nabla V$ is the electric field in a stationary reference frame and **F** is the geomagnetic field. In principle **F** includes the local anomalous magnetic field due to electric current flow, but this contribution is orders of magnitude less than the geomagnetic field, so in practice this perturbation can be ignored and **F** can be taken to be the background geomagnetic field, assumed uniform. The magnetic body force $\mathbf{j} \times \mathbf{F}$ acting on the seawater can be safely ignored because it is minuscule compared to the pressure and buoyancy forces that drive the water motion.

Equation (38) includes the magnetic effect of the moving electrically polarized medium, which is equivalent to that of a magnetized material with magnetization $\mathbf{M} = \mathbf{P} \times \mathbf{v}$. However, the second term in the brackets on the RHS of (38) can be neglected because it is many orders of magnitude less than the conduction current. For example, taking $\sigma = 4\ \mathrm{Sm^{-1}}$, $\varepsilon = 80\varepsilon_0$, $v \le 10\ \mathrm{ms^{-1}}$, and a characteristic length $L \sim$ 1-10 m, implies that the ratio of the contribution to the curl of **B** from motion of the polarized medium to that of the conduction current is less than $(\varepsilon - \varepsilon_0)v/\sigma L \approx 10^{-9}$ - 10^{-10}. Thus we may safely drop the term involving **P** from (38) when determining the electromagnetic fields.

Incorporating seawater motion into the analysis is straightforward, provided the water is assumed to be incompressible (an excellent approximation) and its flow is irrotational, which is an idealization that is reasonable in many circumstances, provided the flow is not too rapid or turbulent. Incompressibility implies that

$$\nabla\mathbf{.v} = 0. \tag{41}$$

For irrotational flow

$$\nabla\times\mathbf{v} = 0, \tag{42}$$

which implies that

$$\mathbf{v} = \nabla\phi, \tag{43}$$

where ϕ is the velocity potential. For initially uniform water flow $\mathbf{v}_0$ the unperturbed velocity potential is $\phi_0 = \mathbf{r.}\ \mathbf{v}_0$. From (54) and (56), ϕ obeys Laplace's equation. For ideal fluid flow around an ellipsoid the boundary value problem for ϕ is identical in form to the problem for $-V$ around an ellipsoidal cavity in a stationary conductive medium. For example, the perturbed velocity potential and corresponding velocity around a spherical cavity are

$$\phi(\mathbf{r}) = \left(1+\frac{a^3}{2r^3}\right)\mathbf{v}_0\mathbf{.r}, \quad (r \ge a) \tag{44}$$

$$\mathbf{v} = \left(1+\frac{a^3}{2r^3}\right)\mathbf{v}_0 - \frac{3a^3(\mathbf{v}_0\mathbf{.\hat{r}})}{2r^3}\hat{\mathbf{r}} \quad (r \ge a). \tag{45}$$

The perturbation of the velocity field is given by:

$$\mathbf{v}' = \mathbf{v} - \mathbf{v}_0 = \frac{a^3}{2r^3}\mathbf{v}_0 - \frac{3a^3(\mathbf{v}_0\mathbf{.\hat{r}})}{2r^3}\hat{\mathbf{r}} \quad (r \ge a). \tag{46}$$

The electric potential associated with flow of a conductive medium is not necessarily harmonic. For this type of ideal fluid flow, however, the electric potential does obey Laplace's equation. From (35) and (40)

$$\nabla^2 V = -\nabla\mathbf{.E} = (\varepsilon-\varepsilon_0)\nabla.(\mathbf{v}\times\mathbf{F})/\varepsilon$$
$$= [(\varepsilon-\varepsilon_0)/\varepsilon][\mathbf{F}.(\nabla\times\mathbf{v}) - \mathbf{v}.(\nabla\times\mathbf{F})] = 0,$$

because **v** is irrotational and **F** is uniform.

Assuming for the moment that there is no applied electric field in the stationary reference frame, from (39) the boundary condition that there is no electric current flow into the cavity implies

$$(\mathbf{j.\hat{r}})_{r=a} = \sigma[\mathbf{E}'.\hat{\mathbf{r}} + (\mathbf{v}\times\mathbf{F}).\hat{\mathbf{r}}]_{r=a} = 0. \tag{47}$$

Therefore

$$-(\mathbf{E}'.\hat{\mathbf{r}})_{r=a} = \left(\frac{\partial V'}{\partial r}\right)_{r=a} = [(\mathbf{v}\times\mathbf{F}).\hat{\mathbf{r}}]_{r=a}, \tag{48}$$

where $\mathbf{E}' = -\nabla V'$ here is the electric field due to the charge distribution on the cavity wall that is established initially by the Lorentz field associated with the moving medium.

From (45) and (48), the boundary condition is

$$\left(\frac{\partial V'}{\partial r}\right)_{r=a} = \frac{3}{2}\hat{\mathbf{r}}.([\mathbf{v}_0 - (\mathbf{v}_0.\hat{\mathbf{r}})\hat{\mathbf{r}}]\times\mathbf{F}) = \frac{3}{2}\hat{\mathbf{r}}.[\mathbf{v}_0\times\mathbf{F}], \tag{49}$$

since $\hat{\mathbf{r}}.[\hat{\mathbf{r}}\times\mathbf{F}] = 0$.

Equation (50) gives a Neumann boundary condition for the external potential. The solution is

$$V'(r \ge a) = -\frac{3\mathrm{a}^3}{4\mathrm{r}^2}\hat{\mathbf{r}}.(\mathbf{v}_0\times\mathbf{F}) = -\frac{3\mathrm{a}^3}{4\mathrm{r}^3}\mathbf{r}.(\mathbf{v}_0\times\mathbf{F}). \tag{50}$$

The electric potential given by (50) is that of an elementary current dipole in a homogeneous medium of conductivity σ with moment

$$\mathbf{p} = -3\pi\sigma a^3\mathbf{v}_0\times\mathbf{F} = -\frac{9\sigma\tau}{4}\mathbf{v}_0\times\mathbf{F}. \tag{51}$$

The internal potential is determined by the Dirichlet boundary condition that matches the potential across the cavity boundary, yielding

$$V'(r \le a) = -\frac{3}{4}\mathbf{r}.(\mathbf{v}_0\times\mathbf{F}). \tag{52}$$

The corresponding internal electric field due to the water flow in the surrounding medium is therefore

$$\mathbf{E}'(r \le a) = \frac{3}{4}(\mathbf{v}_0\times\mathbf{F}). \tag{53}$$

Equation (53) implies that the internal electric field arising from the perturbed water flow is 0.75 times the Lorentz field that pertains to the unperturbed flowing medium.

Differentiating (50) gives the external electric field components due to the water flow around the cavity. Combining these, the anomalous external electric field vector is

$$\mathbf{E}'(r \ \ge\ a) = -\nabla V'(r \ \ge\ a) = \frac{3a^3[\mathbf{v}_0\times\mathbf{F} - 3\hat{\mathbf{r}}.(\mathbf{v}_0\times\mathbf{F})\hat{\mathbf{r}}]}{4r^3}. \tag{54}$$

It is obvious from (53)-(54) that only the component of the unperturbed water flow that is perpendicular to the geomagnetic field contributes to the anomalous electric field in and around the cavity.

The average electric field components between symmetrically placed potential electrodes, as measured by a triaxial electrometer, are

$$\overline{E}_x = \frac{V'(-x,0,0) - V'(x,0,0)}{2|x|} = \begin{cases} \dfrac{3a^3(v_y^0 F_z - v_z^0 F_y)}{4|x|^3}, & (|x| \ge a), \\ \dfrac{3(v_y^0 F_z - v_z^0 F_y)}{4}, & (|x| \le a). \end{cases} \quad (55)$$

$$\overline{E}_y = \frac{V'(0,-y,0) - V'(0,y,0)}{2|y|} = \begin{cases} \dfrac{3a^3(v_z^0 F_x - v_x^0 F_z)}{4|y|^3}, & (|y| \ge a), \\ \dfrac{3(v_z^0 F_x - v_x^0 F_z)}{4}, & (|y| \le a). \end{cases} \quad (56)$$

$$\overline{E}_z = \frac{V'(0,0,-z) - V'(0,0,z)}{2|z|} = \begin{cases} \dfrac{3a^3(v_x^0 F_y - v_y^0 F_x)}{4|z|^3}, & (|z| \ge a), \\ \dfrac{3(v_x^0 F_y - v_y^0 F_x)}{4}, & (|z| \le a). \end{cases} \quad (57)$$

As an example, (55)-(57) show that a favorably oriented water flow speed of 1 ms^{-1} in a geomagnetic field of 60 μT produces a substantial internal electric field of up to 45 $\mu\mathrm{Vm}^{-1}$. As the potential electrodes are moved away from the cavity wall, the measured average field decreases from that value to ~5.7 $\mu\mathrm{Vm}^{-1}$ at $r = 2a$, i.e. when the electrometer baseline is twice the cavity diameter, and to ~1.7 $\mu\mathrm{Vm}^{-1}$ at $r = 3a$. In general an applied electric field is also present and the perturbation of this field by the cavity, which is discussed above, supplements the effects of water flow around the cavity.

There are two contributions to the magnetic effects of flow of a conductive fluid around a measurement capsule. The Lorentz field associated with water flow produces an electric current, which is diverted around the insulating capsule. A magnetic field and associated gradient are produced by this perturbed electric current flow. Additionally, the diversion of water flow around the capsule produces a local perturbation of the Lorentz field, with associated induced currents and a secondary magnetic field. The vortex sources of the magnetic field may be separated into contributions $\mathbf{B}'_E$ from the anomalous conduction current and $\mathbf{B}'_v$ from the perturbation of the water flow, with

$$\nabla \times \mathbf{B}'_E = \mu_0 \sigma \mathbf{E}', \quad (58)$$

$$\nabla \times \mathbf{B}'_v = \mu_0 \sigma \mathbf{v}' \times \mathbf{F}. \quad (59)$$

In (59) the contribution of moving polarized fluid has been neglected, as justified above. The external magnetic field associated with the equivalent current dipole (51) is given by the Biot-Savart law. The result is

$$\mathbf{B}'_E(r \ge a) = \frac{\mu_0(\mathbf{p} \times \mathbf{r})}{4\pi r^3} = \frac{-3\mu_0 \sigma a^3 [(\mathbf{v}_0 \times \mathbf{F}) \times \mathbf{r}]}{4r^3}. \quad (60)$$

The magnetic field lines *due to this effect alone* circulate around an axis of the sphere that is perpendicular to both $\mathbf{v}_0$ and $\mathbf{F}$. The Cartesian components of the magnetic field due to this current dipole are harmonic. The continuation of this field into the cavity is

$$\mathbf{B}'_E(r \ge a) = \frac{-3\mu_0 \sigma [(\mathbf{v}_0 \times \mathbf{F}) \times \mathbf{r}]}{4}. \quad (61)$$

There is an additional contribution to the anomalous magnetic field, however, from the electric current associated with the Lorentz field of the *perturbed* water flow. From (59) it can be seen that, unlike $\mathbf{B}'_E$, the Cartesian components of $\mathbf{B}'_v$ are not harmonic outside the cavity, since

$$\begin{aligned} \nabla \times \nabla \times \mathbf{B}'_v &= \nabla(\nabla . \mathbf{B}'_v) - \nabla^2 \mathbf{B}'_v = -\nabla^2 \mathbf{B}'_v = \mu_0 \sigma \nabla \times (\mathbf{v}' \times \mathbf{F}) \\ &= \mu_0 \sigma [(\mathbf{F}.\nabla)\mathbf{v}' - \mathbf{F}(\nabla . \mathbf{v}') - (\mathbf{v}'.\nabla)\mathbf{F} + \mathbf{v}'(\nabla . \mathbf{F})]. \end{aligned} \quad (62)$$

The last three terms on the RHS drop out by (41) and the fact that $\mathbf{F}$ is uniform. Therefore

$$\nabla^2 \mathbf{B}' = \nabla^2 \mathbf{B}'_v = -\mu_0 \sigma (\mathbf{F}.\nabla)\mathbf{v}'. \quad (63)$$

where $\mathbf{B}'$ is the total anomalous field arising from currents associated with the perturbed electric field and the Lorentz field of the perturbed water flow.

The general solutions of the Poisson equations (63) for the Cartesian components of $\mathbf{B}'$ comprise particular solutions supplemented by harmonic functions. It can be verified by calculating the Laplacian of each of the following equations, and the divergence and curl of the vector formed from all three components, that solutions of (63) that yield the correct divergence and curl everywhere for the total exterior field are

$$\begin{aligned} B'_x = \frac{\mu_0 \sigma a^3 v_0}{4r^5} \Big[& F_x x (2x^2 - y^2 - z^2) \\ & + (F_y y + F_z z)(5x^2 + 2y^2 + 2z^2) \Big], \end{aligned} \quad (64)$$

$$B'_y = \frac{\mu_0 \sigma a^3 v_0}{4r^5}\left[F_x y\left(2x^2 - y^2 - z^2\right) + F_y x\left(y^2 - 2x^2 - 2z^2\right) + 3F_z xyz\right], \quad (65)$$

$$B'_z = \frac{\mu_0 \sigma a^3 v_0}{4r^5}\left[F_x z\left(2x^2 - y^2 - z^2\right) + 3F_y xyz + F_z x\left(z^2 - 2x^2 - 2y^2\right)\right], \quad (66)$$

The radial component of $\mathbf{B}'$ is therefore

$$B'_r = \frac{B'_x x + B'_y y + B'_z z}{r} = \frac{\mu_0 \sigma a^3 v_0}{4r^4}\left[F_x\left(2x^2 - y^2 - z^2\right) + 3F_y xy + 3F_z xz\right]. \quad (67)$$

At the surface of the cavity this becomes

$$B'_r(r = a^+) = \frac{\mu_0 \sigma a v_0}{4}\left[F_x\left(3\cos^2\theta - 1\right) + 3F_y \cos\varphi \sin\theta\cos\theta + 3F_z \sin\varphi\sin\theta\cos\theta\right] \quad (68)$$

Within the cavity there are no currents and $\mathbf{B}'$ is therefore derivable from a harmonic scalar potential Ω, i.e.

$$\mathbf{B}' = -\nabla\Omega, \quad (69)$$

where

$$\nabla^2\Omega = -\nabla.\mathbf{B}' = 0. \quad (70)$$

Matching the radial component of the field across the cavity wall gives the Neumann boundary condition:

$$B'_r(r = a^+) = -\left(\frac{\partial\Omega}{\partial r}\right)_{r=a^-}. \quad (71)$$

The solution of (70) subject to (71) is:

$$\Omega = \frac{\mu_0\sigma v_0}{8} r^2\left[F_x\left(3\cos^2\theta - 1\right) - 3F_y \cos\varphi\sin\theta\cos\theta - 3F_z \sin\varphi\sin\theta\cos\theta\right] = \frac{\mu_0\sigma v_0}{8}\left[F_x\left(2x^2 - y^2 - z^2\right) - 3F_y xy - 3F_z xz\right]. \quad (72)$$

The internal field components, obtained by differentiation of (72), are therefore

$$\left.\begin{aligned} B'_x &= -\frac{\partial\Omega}{\partial x} = \frac{\mu_0\sigma v_0}{8}\left[-4F_x x + 3F_y y + 3F_z z\right] \\ B'_y &= -\frac{\partial\Omega}{\partial y} = \frac{\mu_0\sigma v_0}{8}\left[2F_x y + 3F_y x\right] \\ B'_z &= -\frac{\partial\Omega}{\partial z} = \frac{\mu_0\sigma v_0}{8}\left[2F_x z + 3F_z x\right] \end{aligned}\right\} \quad (r \le a). \quad (73)$$

and the gradient tensor elements are

$$\left.\begin{aligned} B'_{xx} &= \frac{-\mu_0\sigma v_0 F_x}{2} \\ B'_{xy} &= \frac{3\mu_0\sigma v_0 F_y}{8} \\ B'_{xz} &= \frac{3\mu_0\sigma v_0 F_z}{8} \\ B'_{yy} &= \frac{\mu_0\sigma v_0 F_x}{4} \\ B'_{yz} &= 0 \\ B'_{zz} &= \frac{\mu_0\sigma v_0 F_x}{4} \end{aligned}\right\} \quad (r \le a). \quad (74)$$

Since $\mathbf{B}'$ as given by (64)-(66), (73) is continuous, has the correct divergence and curl everywhere, and is regular at the origin and at infinity, it represents the unique solution to the boundary value problem. Note that the internal field components vary linearly with the Cartesian co-ordinates and that the internal gradient is uniform.

The contributions of water flow parallel to the geomagnetic field (which produces no background Lorentz field) and perpendicular to the geomagnetic field produce different configurations of the anomalous electric and magnetic fields in and around the cavity. For water flow parallel to **F**, there is no anomalous electric field. The magnetic field arises solely from the diversion of water flow around the cavity, producing flow components that are not parallel to **F**. The electric currents induced by this flow circulate about the x_1 axis (parallel to $\mathbf{v}_0$ and **F**) and effectively simulate a gradient coil set, with coils in series opposition, wound around the cavity. The magnetic field is rotationally symmetric about $\mathbf{v}_0$ and **F**. Fig.3 shows the resulting magnetic field pattern.

For the component of water flow that is perpendicular to **F** the internal magnetic field lines are confined to planes perpendicular to $\mathbf{v}_0 \times \mathbf{F}$, and the only nonzero gradient tensor element is B_{xy}, if y is taken to be parallel to **F**. The internal magnetic field lines are hyperbolas given by $x^2 - y^2 = \pm C^2$ (see Fig.4). The external field configuration is more complex than for the case of $\mathbf{v}_0$ parallel to **F**, but the overall field fall-off is $1/r^2$ for all cases. From (64)-(66) and (73)-(74), the magnetic effects of seawater flow can be substantial. For example, for a geomagnetic field of 60 μT and a cavity of radius 0.5 m within a 1 ms^{-1} flow, the field perturbation adjacent to the cavity boundary is up to ~75 pT and the gradients are of the order of 150 pT/m.

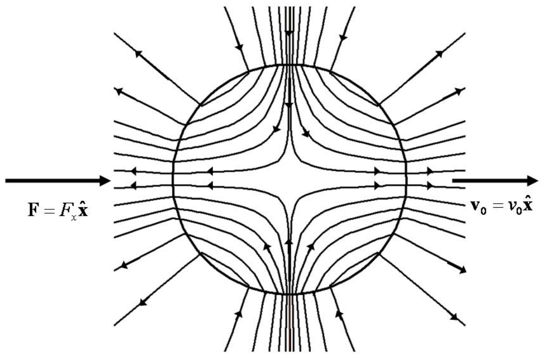

Fig.3. Lines of anomalous magnetic field within and around a spherical cavity due to the water flow component parallel to the geomagnetic field. The view is a section containing $\mathbf{v}_0$ and $\mathbf{F}$, through the centre of the sphere. The field is rotationally symmetric about the x axis. Inside the cavity the gradient is uniform, with maximum gradient parallel to the x axis.

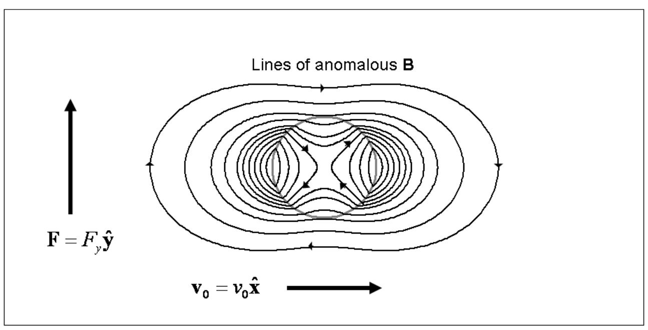

Fig.4. Lines of anomalous magnetic field within and around a spherical cavity due to the water flow component perpendicular to the geomagnetic field. The view is a section containing $\mathbf{v}_0$ and $\mathbf{F}$, through the centre of the sphere. Inside the cavity the only nonzero gradient tensor element is B_{xy}, which is uniform, and gradients perpendicular to the section are zero.

CONCLUSIONS

This study has shown that perturbation of electric and magnetic fields and gradients by the measurement process is significant in the marine environment. Accurate measurements require corrections for diversion of electric currents and water flow around measurement capsules. This has implications for marine magnetotelluric and CSEM measurements and for magnetic tensor gradiometry surveys in the ocean.

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