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Analytic comparison of apparent polar wander paths

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For geological times older than the age of the sea floor and for which we can justifiably extrapolate specific supercontinent configurations, an appeal to palaeomagnetism would seem the most appropriate approach to solve problems of continental distribution. In order to improve upon procedures whereby apparent polar wander paths are matched solely by inspection, we have developed an iterative technique which allows us to match paths on the surface of a sphere on an objective basis. The notion of a minimum combined path length from a comparison of two data sets is used to determine the optimal match-up. The technique is appropriate to cases where the only dating available for the points on each path is their relative chronology.

1. Introduction

The representation of palaeomagnetic data in its general form is often achieved by calculating time-averaged poles, i.e. palaeomagnetic poles, according to the axial-geocentric dipole-field model [1]. A chronologically ordered set of poles obtained from studies relating to a particular continent or craton represents an apparent polar wander path (APWP) for that landmass. By comparing similarly obtained data sets for different continents it has been possible to make specific deductions about their relative displacements during geological time. Graham et al. [2] have described the use of model APW for uniquely determining the relative position of the component parts of a supercontinent. Methods for comparing palaeomagnetic data as a means of describing continental drift have benefitted greatly from the results of fitting continental margin morphologies [3], and from analyses of marine magnetic anomalies and the fracture systems discovered from mapping the sea floor [4].

Prior to the availability of the geometrical fits, palaeomagnetic data from two or more continents were generally fitted by inspection (e.g. [5]). Despite the lack of rigour of the technique, the data pointed to a distinction between specific drift models, but more importantly, they provided the vital information that accelerated the acceptability of the continental drift hypothesis [6,7]. The method of matching continental shelf morphologies, for example at the 1000-m isobath, pioneered by Bullard et al. [3], led to a geometric formalisation of du Toit's [8] reconstruction of the southern hemisphere continents and India through the work of Smith and Hallam [9]. Their reconstruction provided palaeomagnetists with a model supercontinent, arguably realistic on geological grounds, as a basis upon which to test the palaeomagnetic data and to compare apparent polar wander paths from the land masses which comprised the supercontinent.

The results of matching the marine magnetic anomaly patterns across mid-oceanic ridges have further reinforced the reconstruction models of du Toit [8], Bullard et al. [3] and Smith and Hallam [9]. Small adjustments have been made, see for example Norton and Sclater [10], Powell et al. [11], but it is doubtful if the palaeomagnetic method

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currently offers the resolution required to distinguish such subtleties [12].

In order to investigate problems in continental reconstruction and the displacement of landmasses for periods of earth history older than the present sea floor, we are again faced with making interpretations based primarily on the comparison of the palaeomagnetic data. In this regard, the antiquity of the geometrical reconstructions is questioned: for what length of geological time should they be considered to be valid? Several attempts have been made to reconcile palaeomagnetic data (e.g. [13–18]) for Precambrian time but their quality and quantity do not permit an unequivocal solution. The conditions for comparing data within continents and between continents for this part of earth history are analogous to conditions that existed prior to the availability of other quantitative techniques which provided constraints on models for reconstructing continents for Phanerozoic time. Therefore, in order to obviate the need to fit pole paths by inspection, we have developed an iterative technique which allows us to match paths on the surface of a sphere. The data set we use to demonstrate this technique is interesting, if not controversial [15,19], in that it poses a fundamental paradox to those investigating the nature of the lithosphere in the Proterozoic. Nevertheless we emphasize it is the technique we are presenting here.

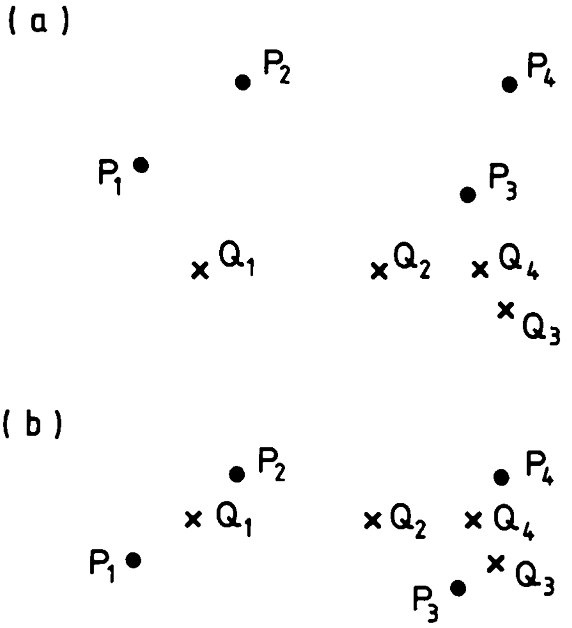

Fig. 1. (a) Initial positions of two paths $P_1, \ldots, P_4$ and $Q_1, \ldots, Q_4$ (each in chronological sequence). (b) Q-path moved relative to P-path in an attempt to reduce the combined path length.

2. The method for matching apparent polar wander paths

To find an optimal match-up of two apparent polar wander paths we have used the notion of *combined path length* (CPL), which is most easily explained by considering two short sequences of points. Consider the two sets of points $(P_1, \ldots, P_4)$ and $(Q_1, \ldots, Q_4)$ positioned as shown in Fig. 1a, where the P's and Q's are each ordered in time. Consider the 8 points as one combined sequence, within which the P's still maintain the time ordering relative to themselves, and similarly for the Q's, for example, the combined sequence $P_1P_2Q_1Q_2Q_3P_3Q_4P_4$. We say that the *length* of this sequence is the sum of the distances $P_1P_2 + P_2Q_1 + \ldots + Q_4P_4$. Clearly there will be at least one combined sequence for which this length is smallest; this length is the *combined path length* for this configuration of the two sequences $(P_1, \ldots, P_4)$ and $(Q_1, \ldots, Q_4)$. For the example in Fig. 1a, the appropriate sequence is $P_1Q_1P_2Q_2P_3Q_3Q_4P_4$.

More generally, suppose that there are mP's and nQ's. Then there are $\binom{m+n}{n}$ possible combined sequences in which the time-integrity of each component sequence is maintained. To find the CPL for a particular configuration of the sequences requires computation of many, if not all, of the $\binom{m+n}{n}$ lengths associated with these sequences, which is not feasible for many practical situations. However, an approximation to the CPL can be obtained as follows.

Starting at P_1, say, look at all possible sequences using a total of s points from the two paths (e.g. $P_1Q_1Q_2P_2P_3 \ldots P_{s-2}$), with the points selected from a given path being chosen in sequence order. Of these 2^{s-1} possible sequences, choose the one with smallest path length. The second point of this sequence is either P_2 or Q_1. Now move to this point, Q_1 say, and look at all possible sequences using a total of s points from $P_2, \ldots, P_m$ and $Q_2, \ldots, Q_n$, and so on. The sequence finally obtained when all points are exhausted is called the approximate CPL based on *search distance* s, or CPL(s). It is necessary to perform these calculations with 4 different starting points, namely P_1, Q_1, P_m and Q_n. (For P_m or Q_n as the starting point, the paths are fitted backwards in time.)

The consequent reduction in computation is from $\binom{m+n}{n}$ paths to rather fewer than $(m+n-s)2^{s+1}$ paths. For the orders of magnitude of m and n considered in this paper (20), it is feasible to use a search distance $s = 12$ on the CDC Cyber 7600.

This notion is readily applied to the problem of finding the optimal match-up of two apparent polar wander paths, when the only dating available for the points on each path is their relative chronology. The relative chronology of points between paths is assumed unknown (although the technique is readily adapted to situations where some between-path datings are available). The aim is to find that match-up, or configuration of the two sequences, which yields the minimal CPL. Thus, the following iterative process is required. Let s be the chosen search distance. Then:

(1) For a given starting configuration of the two paths (e.g. as in Fig. 1a) calculate CPL(s).

(2) Move one path relative to the other in an attempt to improve the match-up.

(3) Calculate the CPL(s) for this new config-

TABLE 1

Palaeomagnetic data sets for North America and Africa. "Africa rotated" has been obtained following optimal match-up (using the Euler pole for $s = 8$, Table 3, A)

North America			Africa			African rotated
	rock unit/formation	palaeomagnetic pole (lat., long.)		rock unit/formation	palaeomagnetic pole (lat., long.)	palaeomagnetic pole (lat., long.)
1	Coleman Member (Gowganda)	60N, 080E	1	Upper Ventersdorp Lavas	71N, 173E	63N, 246E
2	Firstbrook Member (Gowganda)	67N, 158E	2	Transvaal Lavas	40N, 194E	35N, 217E
3	Otto Stock	69N, 227E	3	Tarkwaian Intrusions	53N, 216E	37N, 242E
4	Big Spruce Complex	67N, 247E	4	Syntectonic Igneous Rocks	55N, 226E	35N, 249E
5	Mugford Volcanic Series	49N, 217E	5	Obusai Dolerite dyke	56N, 248E	31N, 264E
6	Abitibi Dykes	30N, 225E	6	Aftout Diorite + dykes (1)	55N, 255E	29N, 267E
7	Otish Gabbro	35N, 253N	7	Obusai Greenstone	50N, 282E	21N, 284E
8	Nipissing Diabase	42N, 258E	8	Orange River Lavas	19N, 254E	6S, 256E
9	Spanish River Complex	37N, 264E	9	Aftout Diorite + dykes (2)	20N, 260E	6S, 262E
10	Gunflint Formation	28N, 266E	10	Aftout Diorite + dykes (3)	10N, 260E	16S, 260E
11	Indin Dykes	19N, 284E	11	Ivory Coast Dolerite	11N, 282E	18S, 283E
12	Dubawnt Group	07N, 277E	12	Cunene Anorthosite, Angola	3S, 255E	22S, 253E
13	Upper Gibraltar	08N, 285E	13	Aftout Diorite + dykes (4)	5N, 220E	7S, 220E
14	McLeod Formation	05N, 290E	14	Lower Venterdorp Lavas	4N, 220E	8S, 220E
15	Martin Formation	09S, 288E	15	Aftout Diorite + dykes (5)	0, 230E	16S, 227E
16	Nipissing Diabase	15S, 264E	16	Aftout Diorite + dykes (6)	5S, 230E	20S, 225E
17	Douglas Peninsula	17S, 258E	17	Aftout diorite + dykes (7)	11S, 230E	26S, 222E
18	Takiyuak Formation	14S, 249E	18	Bushveld Gabbro Complex	23S, 216E	30S, 202E
19	Stark Formation	15S, 215E	19	Vredefort Ring Complex	22S, 207E	25S, 195E
20	Tochatwi Formation	18S, 218E	20	Losberg Intrusion	33S, 216E	38S, 196E
			21	Aftout Gabbro	29S, 235E	44S, 217E

For North American pole positions see references 14, 21, 22 and for African, references 23, 24. Poles are ordered according to the sequence shown in Fig. 2.

uration. If it is smaller, this match-up is better (e.g. Fig. 1b will provide a better match-up in terms of CPL(s) than Fig. 1a).

(4) Go to 2.

Eventually, a stage is reached where any reduction in CPL(s) is negligible. Note that it is possible for different configurations to have the same minimal CPL. They must be distinguished on scientific grounds.

For APWP's, step 2 can be done using standard iterative procedures such as the Nelder-Mead simplex method [20] to estimate the parameters (θ, ϕ, ω) of the rotation providing this optimal match-up. Initially, a small search distance ($s = 2$, 3 or 4) can be employed to obtain rough estimates of θ, ϕ and ω, and then a somewhat larger (and hence more computationally expensive) value used to refine the estimate.

3. Results

The technique has been applied to the APWP's for North America and Africa for the approximate time interval 2300 m.y. to 1900 m.y. and to the APWP's for North America and Australia for the approximate time interval 1800 m.y. to 1600 m.y. The data sets and APWP models used are those used by Embleton and Schmidt [15]. Names of rock units and their corresponding pole positions are listed in Tables 1 and 2. In each case the North American APWP has been held fixed and the complementary data sets rotated to provide a best fit according to the procedure outlined. Euler poles obtained following a series of iterations are listed in Table 3.

Good agreement between the data sets for North America and Africa has been achieved using search

TABLE 2

Palaeomagnetic data sets for North America and Australia. The Australian palaeomagnetic poles have been rotated following optimal match-up ($s = 6$, Table 3, B)

North America			Australia			Australian rotated palaeomagnetic pole (lat., long.)
	rock unit/formation	palaeomagnetic pole (lat., long.)		rock unit/formation	palaeomagnetic pole (lat., long.)	
1	Pensinsula Sill	22S, 263E	1	Mount Tom Price, iron ore	22S, 237E	35S, 269E
2	Pearson Formation (A)	19S, 283E	2	Mount Newman, iron ore	17S, 246E	28S, 276E
3	Flin-Flon greenstone (B)	21S, 310E		Hart Dolerite	(29S, 226E)	(46S, 260E)
4	Eskimo Volcanics	40S, 2E	3	Mount Goldsworthy, iron ore	20S, 264E	24S, 295E
5	Group E [1]	9N, 1E		Koolyanobbing, Dowd's Hill,		
6	Et Then	1S, 312E		iron ore	(26S, 272E)	(26S, 305E)
7	Sparrow Dykes	12E, 291E	4	Pilbara dyke, ENE trending	28S, 272E	28S, 306R
8	Pearson Formation (C)	6N, 283E	5	Yilgarn dykes, F group	25S, 282E	21S, 313E
9	Melville Daly Bay	11N, 259E	6	Pilbara dykes, NW trending	32S, 328E	13S, 355E
10	Flaherty Volcanics	0, 244E		Edith River Volcanics	(6N, 14E)	(29N, 36E)
11	Haig Intrusives	1N, 247E		Yilgarn Dykes, A group	(22N, 314E)	(34N, 325E)
12	Eskimo overprint	19N, 243E	7	Iron Monarch, positive	15N, 272E	12N, 288E
13	Seton Formation (B)	25N, 263E	8	Gawler dykes, B group	23N, 266E	17N, 297E
14	Flin-Flon greenstones (A)	27N, 265E		Mount Goldsworthy, iron ore	(22N, 259E)	(13N, 274E)
15	Seward Formation (C)	53N, 248E	9	Yilgarn dykes, D group	24N, 226E	5N, 244E
			10	Iron Prince	39N, 247E	25N, 257E
			11	Mt. Isa province dykes	53N, 282E	49N, 275E
			12	Iron Monarch, negative	64N, 267E	53N, 254E
			13	Gawler dykes, A group	61N, 231E	42N, 235E
			14	Gawler Range Volcanics	60N, 230E	41N, 235E

Bracketed results are those with $A_{95} > 15°$, and have not been used in the iteration procedure. Results for North America have been taken from references 21, 22 and for Australia from references 15, 17, 24. Poles are ordered according to the sequence shown in Fig. 3.

TABLE 3

Rotation parameters calculated for varying search distances

Search distance (s)	θ_p (°)	ϕ_p (°)	ω (°)	Combined minimum path length
A. APW match-up for North America and Africa				
4	−53.4	155.0	+22.3	8.413
6	−7.1	182.8	+68.4	8.968
8	1.3	16.1	+29.0	7.880
10	10.5	18.4	+22.1	7.980
12	9.0	17.2	+21.0	7.970
B. APW match-up for North America and Australia				
2	81.7	182.9	+16.8	8.265
4	71.0	286.2	+29.4	7.114
6	42.3	288.0	+31.4	6.974
8	71.1	305.1	+23.8	7.131
10	57.6	294.1	+29.5	7.066

Search distance (s) and combined minimum path length (CPL) are defined in section 2.

Parameters which define the Euler pole are θ_p (latitude), ϕ_p (longitude) and ω (rotation angle). Negative angles represent southern hemisphere latitudes.

distances (s) of 8, 10 and 12. An optimal match-up based on the concept of minimum path length is revealed for $s = 8$ (see Table 3, A). Visually, there is little difference between this case and for $s = 10$ or 12. Optimal match-up of the Australian data set and the North American data set for the younger interval of time is achieved for $s = 6$ (Table 3, B). In this case the pole positions used in the analysis were those whose semi-angles of cones of confidence (α_{95}) were $\leqslant 15°$. Again, a visual appraisal revealed little difference between any of the solutions. Examples of match-up for $s = 4$ and 10 are shown in Figs. 2 and 3 for the respective continental pairs.

The recognition of common apparent polar wander during the early Proterozoic was made on the basis of a visual comparison of these data sets. Those findings remain essentially valid. The Euler poles for Africa and Australia with respect to North America lie within, or close to, the continents, suggesting that primarily an azimuthal adjustment improves APWP match-up. The rotated

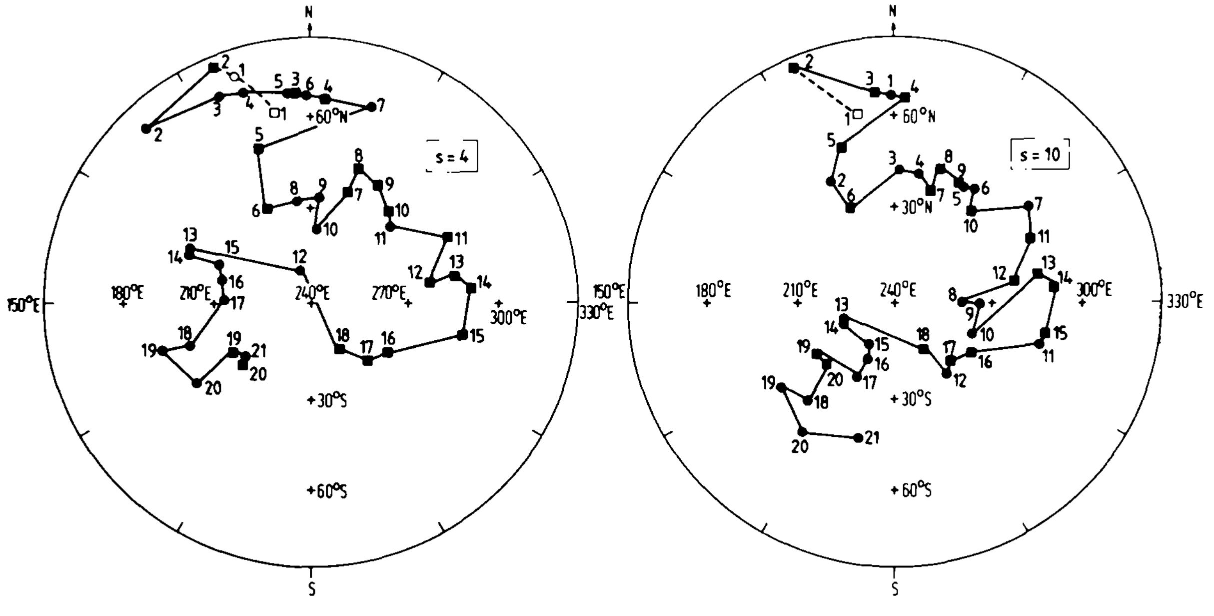

Fig. 2. Match-up between data sets for North America and Africa (see text) obtained for search distances (s) = 4 and 10. The palaeomagnetic poles are plotted on an equal angle (Schmidt) stereographic projection. African data, circles; North American data, squares; open symbols plot on the obscured hemisphere. The pole positions are linked according to the path sequence.

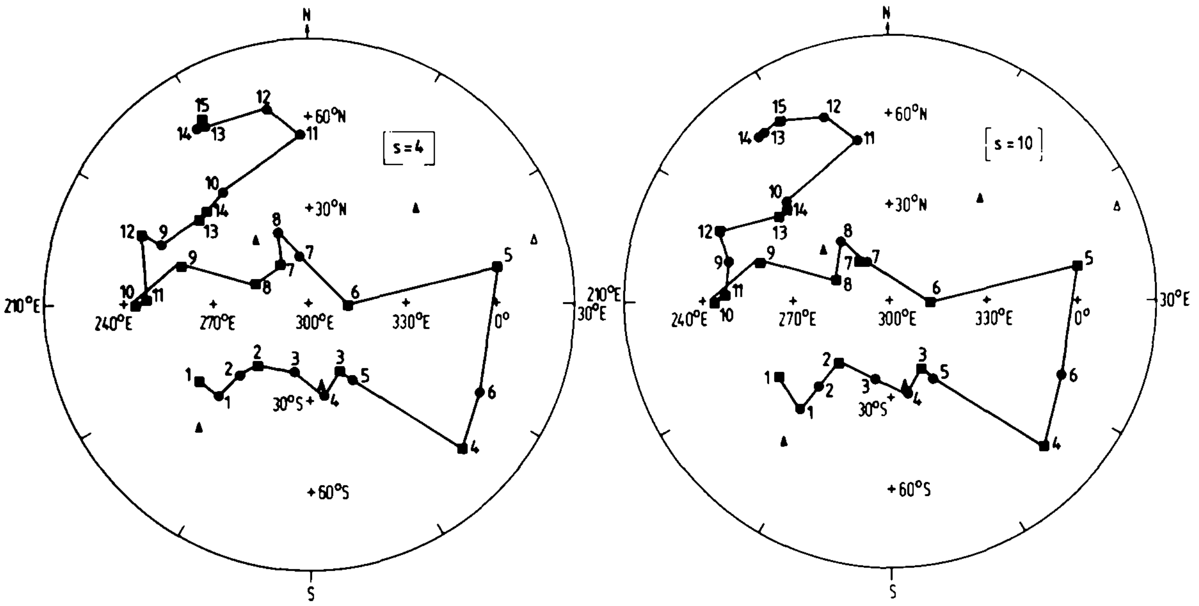

Fig. 3. Match up between data sets for North America and Australia (see text) obtained for search distances (s) = 4 and 10. Squares, North America; circles, Australia; triangles, Australia (not used in the analysis, see Table 2). Open symbols plot on the obscured hemisphere. Pole positions are linked according to the path sequence.

palaeomagnetic pole positions for Africa and Australia are listed in Tables 1 and 2 respectively.

4. Conclusions

The procedure outlined provides us with a convenient objective method for matching two sequences of data points distributed on a sphere. It is applicable to cases such as apparent polar wander paths where only the relative chronology for the points is known. In the examples used, we are of the opinion that this condition is the most appropriate one on which to base those comparisons. The recognition of common signatures in published APWP's for the respective pairs of continents [15] led to the suggestion that to reconstruct continents for the early-mid Proterozoic was unnecessary. Commenting on these conclusions and models of reconstruction proposed by others [21,25], Irving and McGlynn [19] sought to question the validity of the palaeomagnetic data for North America which earlier provided the basis for APWP construction [26]. An appeal to, "some as yet unstudied process", [19, p. 582] for example, as an explanation for magnetisation directions once considered primary is convenient but has created confusion [27]. Absolute chronology of rock units, actual age of magnetisation, measured magnetic directions and measured radiometric ratios are too often poorly correlated. We have preferred to adopt the notion that, within approximate intervals of time, track shape and length are the appropriate features on which to base APWP comparisons.

One unsatisfactory aspect of the work is that it is not computationally feasible to calculate the complete combined path length for data sets of moderate size. The data sets analysed herein required up to 800 seconds of CPU time on the Cyber 7600 to obtain approximate solutions. Investigation is currently under way to find a more efficient computational procedure.

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