# Primer of Use and Interpretation of Signed Digraphs and Bayes Nets

July 2020

Jeffrey M. Dambacher, CSIRO

## Purpose

This primer is to assist a user to run the full models produced for the FRDC as part of the project “Informing development of the 2020-2025 FRDC RD&E plan (FRDC 2018-197)”, and was downloaded from https://research.csiro.au/oceanfutures/frdc-futures/

## Background

Our analysis consists of the method of signed digraphs, their qualitative analysis, and their representation in Bayes nets. We demonstrate these basic methods through an example system of three variables, but the principles are the same for systems of any size.

In the example model of Figure 1, a variable of interest, called “value”, consumes a resource variable from which it derives a rate of reproduction. This same resource variable can increase the occurrence of fire, which has a direct negative effect on the value.



Figure 1. Example signed digraph model of a value-resource-fire ecosystem in which a desired value is sustained by a resource (Res), both of which are regulated by fire. Graph links ending in an arrow denote positive direct effects and links ending in a filled circle denote negative direct effects.

We next assess qualitative model prediction to determine which parts of the system respond predictably to a perturbation. Table 1 presents qualitative prediction for an increase to the resource (say through a supply of nutrients), and an increase in the occurrence of fire (say through an addition of fuel). In the first perturbation scenario both the resource and the occurrence of fire are predicted to increase, while the predicted response of the value is ambiguous, as it could conceivably either increase or decrease depending on the relative strength of the links in the two pathways leading from the resource to the value. In the second perturbation scenario there is a predicted increase in the occurrence of fire and a predicted decrease in the value, while the predicted response of the resource variable is now ambiguous. In this system fire presents itself as indicator that has an interpretable set of predictions across multiple perturbation scenarios, while the other two variables yield ambiguous response prediction.

Table 1. Qualitative predictions of response to two perturbation scenarios for value-resource-fire model (Figure 1).

| Model variable | Increased resource | Increased fire |
| --- | --- | --- |
| Value | ? | – |
| Resource | + | ? |
| Fire | + | + |

We next demonstrate how the qualitative mathematical models can be used in a Bayesian framework to make the same predictions of perturbation response as in Table 1, but also to test the models, diagnose the most likely source of input or driver in a system (i.e., identify the variable through which a perturbation likely entered the system), and to identify the most informative set of indicators with respect to specific variable of interest. Below we have embedded the qualitative dynamics of the value-resource-fire model into a Bayes net. Figure 2 shows a Bayes net with three different types of nodes. In the top tier a single node holds the options for different model structures, which includes one or more signed digraph models and a null model. The null model essentially allocates equal probability to any modelling outcome and provides a relative comparison for allocations of likelihood in model predictions and diagnosis of inputs. The middle tier has nodes that represent the predicted or observed response of the model variables. Note that the links between the nodes in the Bayes net do not mean the same thing as in the signed digraph model. Links in a signed digraph represent direct effects from processes, as in a predator-prey relationship, whereas links within a Bayes net represent statistical dependence. The bottom tier of nodes represents the variables that are potential sources of inputs or perturbations to the system.



Figure 2. Bayes net representation of signed digraph model in Figure 1.

In this configuration the Bayes net can perform four basic functions: it can be used to predict response of system to an input or perturbation (Figure 3), test a models level of consistency with observed responses (Figure 4), diagnose most likely source of an input (Figure 5), and identify most informative indicators (Figure 6).

## Model use 1: Predicting response of system to an input or perturbation

In Figure 3, the Bayes net shows qualitative response predictions where there has been a 100% likelihood for an increase in nutrients, a zero probability for any input to fire, a 100% likelihood for the example model i to be correct, and zero likelihood for the null model. In this configuration, the middle row of observation variables is left unconstrained, and their resulting probabilities give the same response predictions of Table 1 (*i.e.*, a prediction of an increase to resource and fire, and an ambiguous response for the value).



Figure 3. Selecting alternative model and source and direction of input (increase, unchanged, decrease) provides predictions of qualitative response of model variables.

## Model use 2: Which model is most likely?

In Figure 4 a test of the model is performed by selecting the source of the input, here an increase to fire and no change to the resource and entering at least one observation for a variable in the model system. A selection of an observed increase in the value variable shows that this observation is highly inconsistent with predictions of the signed digraph model, which predicted a decrease in this variable. Here the null model, which represents completely ambiguous predictions, is shown to be more consistent with the observed response than the signed digraph model of Figure 1.



Figure 4. Selecting observed response of model variables and source and direction of input (increase, unchanged, decrease) tests consistency of model predictions with observations.

## Model use 3: Identifying Drivers

In Figure 5, the Bayes net is used to diagnose the most likely source of an input (driver) to the system by constraining the alternative model node and each of the observation nodes in the middle row. Here the probabilities expressed in the unconstrained input nodes reveals the likelihood that they are responsible for the observed changes in the state of the system. Thus, an observation of an increase to the value, no change in the resource and a decrease in the occurrence of fire is highly consistent with a negative input having occurred to fire.



Figure 5. Selecting alternative model and observed response of model variables (increase, unchanged, decrease) provides diagnosis of likely source of input to the system.

## Model use 4: Identifying informative indicators for monitoring

Lastly, the Bayes net can be used to identify informative indicators for monitoring through a sensitivity analysis that determines which variables and inputs most influence a variable of interest. In the demonstration of a sensitivity analysis in Figure 6a, alternative model i is allocated 100% likelihood and then the top portion of the value node is selected (denoted by black background in node), and all other observation nodes and input nodes are left unconstrained. From **Menu** select **Network-Sensitivity to Findings**; the resulting analysis (Figure 6b) indicates that for the combined purposes of prediction and diagnosis, the two most informative indicators besides the value node itself are the Input to Fire node (37.6%) and Fire node (21.9%), which together account for 59.5% of mutual information (this in comparison to the information content of the value node itself, which is standardized to 100%). By comparison, the knowledge about the resource or inputs to the resource provide only a minor about of mutual information (5.16%).

(a) 

(b) 

Figure 6. Sensitivity analysis in the value-resource-fire model (Figure 1) to identify most informative nodes to learn about the value node (darkened) for purposes of prediction and diagnosis.

**The models developed for the FRDC can be manipulated in the same way as in the above examples.**