RECEIVER-TRANSMITTER PAIR SELECTION IN MIMO PHASED ARRAY RADAR

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ABSTRACT

The increase in the number of degrees of freedoms (DoF) that is afforded by multiple-input-multiple-output (MIMO) phased arrays is accompanied by an increase in hardware and computational costs. We mitigate this problem in a collocated MIMO phased array system by employing a selection strategy where a subset of K transmitter-receiver (Tx-Rx) pairs is chosen from the available N pairs. We formulate the selection task as an optimization problem using the spatial correlation coefficient (SCC). Minimizing the SCC leads to an increase in the orthogonality of the signal and interference subspaces. We formulate and solve both the joint Tx-Rx selection problem and factored selection where the Tx and Rx are decoupled and treated separately. We show that both approaches can achieve excellent trade-off between performance and cost. While the factored problem compromises performance with respect to the joint Tx-Rx selection, it allows for better transmit power efficiency, thus increasing the received signal-to-noise ratio.

Index Terms— MIMO Radar, Adaptive array beamforming, antenna selection, convex optimization

1. INTRODUCTION

Multiple-input-multiple-output (MIMO) phased array radar, [1], continues to be the subject of intense research. A MIMO phased array transmits a set of noncoherent orthogonal waveforms that is extracted at the receiver by a set of matched filter banks. The diversity in the waveforms provides additional degrees of freedom (DoFs) with respect to standard phased arrays and leads to superior performance in terms of detection, spatial resolution, and parameter identifiability [2, 3].

Spacetime-adaptive processing (STAP) for phased arrays enables the design of optimum space-time adaptive filters to mitigate the effects of clutter and jamming signals [4, 5]. Full dimension STAP, however, is plagued by high computational cost and slow convergence due to the severe requirements on training data for the filter design. A number of dimensionality reduction approaches have been proposed over the years to tackle these issues. Low-rank and reduced-dimension techniques rely on the low-rank property of the interference signals to reduce the dimensionality of the covariance matrix [6, 7]. The beamformer is then designed by a reduced dimensional filter. Single snapshot and hybrid approaches, e.g. [8], improve the convergence by alleviating or eliminating altogether the need for training data. They do not however address the computational complexity problem. Alternatively, dimensionality reduction can be achieved by approaching the problem from the compressive sensing and sparsity-awareness perspective [9].

Antenna selection strategies, on the other hand, have been proposed to simultaneously reduce the hardware and computational cost while preserving the performance [10, 11]. Assuming the number of front-ends (or channels) is limited, a switching scheme was proposed to select a subset of K of the available N antennas. This is done to maximize the separability between a desired signal subspace and an interference subspace. These approaches have been extended to antenna-pulse selection in STAP [12].

STAP methods are also applicable in MIMO phased array by extending the datacube to include the extra dimension generated by the orthogonal waveforms [1]. Although, this gives an increase in the rank of the jammer and clutter subspace, the application of STAP is more challenging and computationally expensive [13], and dimensionality reduction becomes more critical. To this end, a sparsity aware algorithm has been proposed for target localization and DOA estimation in collocated MIMO phased array systems in [14]. The authors propose a random array architecture in which a low number of transmit/receive elements are randomly distributed over a large aperture. However, the antenna placement is then fixed and a potential DoF is not exploited. Sparsity has also been used in distributed MIMO phased array in [15], but the method still involves all measurements.

In this paper, we develop an approach to reduce both the measurements and processing requirements in a collocated MIMO phased array system using a transmitter-receiver (Tx-Rx) pair selection strategy. For a MIMO system with M transmitters and N receivers, the Tx-Rx pair selection involves selecting a subset of K out of the available MN pairs to maximize the separation between a desired and parasitic directions of arrival (DOAs). We formulate this problem as a non-convex optimization and obtain solutions using relaxation methods. However carrying out the selection at the matched filter end requires the use of all transmit and receive channels. Therefore, we present a factored selection strategy where the Tx and Rx selection problems are decoupled and solved separately. Although this reduces the solution space with respect to the joint selection problem, it does provide a more efficient scheme for the radar power utilization and allows for the SNR to be enhanced. Finally, we demonstrate through simulations that both selection strategies significantly reduce computation and hardware costs while maintaining a performance that is comparable to that of the full array.

The remainder of the paper is organized as follows. In section 2 we formulate the Tx-Rx selection problem for MIMO arrays. In section 3 we discuss the performance of the joint and factored selection approaches in terms of cost and power utilization. Section 4 presents some simulation examples to illustrate the performance of the proposed strategies. Finally, some conclusions are given in section 5.

2. RECONFIGURABLE MIMO PHASED ARRAY

The ability of phased arrays to simultaneously steer a beam toward a signal of interest and many nulls in specific (interfering) directions is determined by the spatial correlation coefficient (SCC) [16]. The SCC has a direct relationship to the output signal to interference plus noise ratio (SINR), which is dependent on the generalized inner product of the steering vectors of the signal and interference [17]. The generalized inner product, and hence the SCC, can be interpreted as the cosine of the angle between the signal and interference subspaces. An SCC that is 0 implies that the signal and interference are mutually orthogonal. Thus, minimizing the SCC leads to an enhanced ability of the array processing algorithms to carrying interference nulling.

In MIMO systems the selection is a multidimensional optimization problem on the matched filters (that is Tx-Rx pairs). Below, we formulate both the joint Tx-Rx selection and factored Tx and Rx version and discuss their properties.

2.1. Joint Tx-Rx selection

Consider a MIMO phased array with M transmitters and N receivers. The system employs M mutually orthogonal waveforms at the transmitter and a corresponding set of M matched filters at each receiver element. The contributions of all transmitter and receiver pairs give a corresponding virtual array and an associated increase in the number of DoFs. The locations of the virtual elements are given by the convolution of the locations of the transmitters and receivers [18].

Let the location of the transmitters $\mathbf{P}_\mathrm{T}$ and receivers $\mathbf{P}_\mathrm{R}$ be defined as

$$\mathbf{P}_\mathrm{T} = \begin{bmatrix} x_{\mathrm{T},1} & y_{\mathrm{T},1} \\ x_{\mathrm{T},2} & y_{\mathrm{T},2} \\ \vdots & \vdots \\ x_{\mathrm{T},M} & y_{\mathrm{T},M} \end{bmatrix}, \quad \mathbf{P}_\mathrm{R} = \begin{bmatrix} x_{\mathrm{R},1} & y_{\mathrm{R},1} \\ x_{\mathrm{R},2} & y_{\mathrm{R},2} \\ \vdots & \vdots \\ x_{\mathrm{R},N} & y_{\mathrm{R},N} \end{bmatrix}. \tag{1}$$

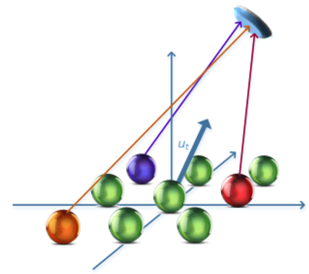

Fig. 1. Collocated MIMO phased array example.

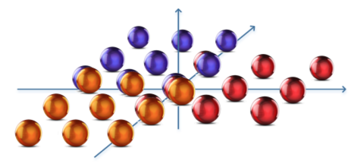

Fig. 2. MIMO virtual array

Then the positions of the MN virtual array elements are

$$\mathbf{P}_\mathrm{V} = \mathbf{P}_\mathrm{R} \otimes \mathbf{1}_M + \mathbf{1}_N \otimes \mathbf{P}_\mathrm{T}, \tag{2}$$

where $\mathbf{1}_N$ is a length-N vector with elements equal to 1, and $\otimes$ is the Kronecker product. An example of a collocated MIMO phased array is shown in Fig. 1. The array includes 3 transmitters and 9 receivers positioned on a uniform 3×3 grid. The virtual array for this configuration is depicted in Fig.2. Let (ϕ_s, θ_s) and (ϕ_j, θ_j) be the DOAs (azimuth and elevation) of the desired signal and interference respectively. The steering vectors of the signal and interference are

$$\mathbf{v}_s = e^{j\frac{2\pi}{\lambda}\mathbf{P}_\mathrm{V}\mathbf{u}_s}, \mathbf{v}_j = e^{j\frac{2\pi}{\lambda}\mathbf{P}_\mathrm{V}\mathbf{u}_j}, \tag{3}$$

where

$$\mathbf{u}_i = [\sin\theta_i \cos\phi_i \quad \sin\theta_i \sin\phi_i]^T. \tag{4}$$

The spatial correlation coefficient, which gives the degree of orthogonality between the signal and noise subspaces is defined as

$$\alpha_{js} = \frac{\mathbf{v}_j^H \mathbf{v}_s}{\|\mathbf{v}_j\| \, \|\mathbf{v}_s\|} = \frac{\mathbf{v}_j^H \mathbf{v}_s}{\sqrt{\mathbf{v}_j^H \mathbf{v}_j}\sqrt{\mathbf{v}_s^H \mathbf{v}_s}} = \frac{\mathbf{v}_j^H \mathbf{v}_s}{MN}. \tag{5}$$

We now introduce a selection vector $\boldsymbol{c}$ that specifies whether a particular element is selected or not. Thus, the i-th element, c_i takes on a value of 1 if the corresponding element is selected, and 0 otherwise. Assuming K out of MN matched filters are active, SCC squared can be written as

$$|\alpha_{js}|^2 = \frac{\mathbf{c}^T \mathbf{W}_r \mathbf{c}}{K^2}, \tag{6}$$

where $\mathbf{W}_r$ is expressed as

$$\mathbf{W}_r = \text{real}(\mathbf{v}_{js}\mathbf{v}_{js}^H), \quad (7)$$

and

$$\mathbf{v}_{js} = \mathbf{v}_s \odot \mathbf{v}_j^H. \quad (8)$$

This can be interpreted as the selection of a subset of elements from the virtual array, or equivalently a subset of Tx-Rx pairs or matched filters within the MIMO array. Recall that the goal here is to minimize the SCC and enhance the separation between the signal and interference subspaces. Thus, the joint Tx-Rx selection problem can be expressed as

$$\begin{aligned} &\min_{\boldsymbol{c}} \ |\alpha_{js}|^2 \\ &\text{s.t. } c_i(c_i - 1) = 0 \quad i = 1...MN, \\ &\text{and } \boldsymbol{c}^T\boldsymbol{c} = K. \end{aligned} \quad (9)$$

This binary selection problem is a non-convex optimization and is known to be NP-hard. Therefore we use two relaxation methods, specifically the Lagrange dual (LD) and direct semidefinite programming (SDP), to obtain lower bounds on the SCC [10].

2.2. Factored Tx and Rx selection

The joint selection problem puts constraint only on the number of output matched filters. Since any subset of filters is a possible solution, the entire set of transmitters must necessarily transmit their waveforms. When the matched filters corresponding to a particular transmit element are not used, this leads to wasted transmit power. This problem can be avoided by factorizing the selection problem into transmit and receive sub-problems. Suppose that we select K_{T} out of M transmit elements and K_{R} out of the available N receive elements. The overall selection vector is then $\boldsymbol{c} = \mathbf{c}_{\text{T}} \otimes \mathbf{c}_{\text{R}}$, where, $\boldsymbol{c}_{\text{R}}$ and $\boldsymbol{c}_{\text{R}}$ are the selection vectors for transmitters and receivers, receptively. The factored SCC becomes

$$|\alpha_{js}|^2 = \frac{(\mathbf{c}_{\text{T}} \otimes \mathbf{c}_{\text{R}})^T \mathbf{W}_r (\mathbf{c}_{\text{T}} \otimes \mathbf{c}_{\text{R}})}{(K_{\text{T}} \times K_{\text{R}})^2}. \quad (10)$$

Thus, the factored Tx/Rx selection problem can be expressed as

$$\begin{aligned} &\min_{\mathbf{c}_{\text{T}},\mathbf{c}_{\text{R}}} |\alpha_{js}|^2 \\ \text{s.t. } &c_{\text{T},i}(c_{\text{T},i} - 1) = 0 \quad i = 1...M, \\ &c_{\text{R},i}(c_{\text{R},i} - 1) = 0 \quad i = 1...N, \\ &\boldsymbol{c}_{\text{T}}^T \boldsymbol{c}_{\text{T}} = K_{\text{T}}, \\ &\boldsymbol{c}_{\text{R}}^T \boldsymbol{c}_{\text{R}} = K_{\text{R}}. \end{aligned} \quad (11)$$

Now we factorize $\mathbf{W}_r$ as

$$\mathbf{W}_r = \mathbf{W}_{r,\text{T}} \otimes \mathbf{W}_{r,\text{R}}, \quad (12)$$

Using the properties of the KronecKer product, the numerator of Eq.10 is rewritten as

$$\begin{aligned} (\mathbf{c}_{\text{T}} \otimes \mathbf{c}_{\text{R}})^T \mathbf{W}_r (\mathbf{c}_{\text{T}} \otimes \mathbf{c}_{\text{R}}) &= (\mathbf{c}_{\text{T}} \otimes \mathbf{c}_{\text{R}})^T (\mathbf{W}_{r,\text{T}} \otimes \mathbf{W}_{r,\text{R}})(\mathbf{c}_{\text{T}} \otimes \mathbf{c}_{\text{R}}) \\ &= (c_{\text{T}}^T \mathbf{W}_{r,\text{T}} \otimes c_{\text{R}}^T \mathbf{W}_{r,\text{R}})(c_{\text{T}} \otimes c_{\text{R}}) \\ &= (c_{\text{T}}^T \mathbf{W}_{r,\text{T}} c_{\text{T}})(c_{\text{R}}^T \mathbf{W}_{r,\text{R}} c_{\text{R}}). \end{aligned} \quad (13)$$

Thus the factored SCC in Eq.10 can be expressed as a multiplication of two SCCs with respect to transmitters and receivers,

$$\begin{aligned} |\alpha_{js}|^2 &= \frac{(c_{\text{T}}^T \mathbf{W}_{r,\text{T}} c_{\text{T}})}{K_{\text{T}}^2} \times \frac{(c_{\text{R}}^T \mathbf{W}_{r,\text{R}} c_{\text{R}})}{K_{\text{R}}^2} \\ &= |\alpha_{js,\text{T}}|^2 \times |\alpha_{js,\text{R}}|^2, \end{aligned} \quad (14)$$

and the selection can be solved as two separate sub-problems.

3. DISCUSSION

The factored Tx-Rx selection operates on a subspace of solutions that is included in the joint Tx-Rx optimization. Therefore, the factored problem reduces the search space and hence computational cost of obtaining the solution, but may not achieve the global solution of the joint problem. However, selecting a subset of transmitters allows the available total transmit power to be allocated only to the chosen elements. This is in contrast to the joint selection problem where all transmitters must be operational to guarantee that all matched filters are available for selection. Thus, assuming a total available transmit power $P_e = P_T$, the transmit power per element in the factored case is $P_e = P_T/K_T$ as opposed to P_T/M for the joint selection case. To illustrate the effect this has on the output SINR, we consider a scenario where a total transmit power P_T is available. Now the power received by the m-th matched filter in the n-th receiver is $P_{n,m} = \alpha P_e$, where α represents the channel gain (including target cross-section.) Assuming that the interference is much stronger than the noise, the output SINR can be expressed as [10]:

$$SINR_{\text{out}} = \alpha K \frac{P_e}{\sigma^2} \left(1 - |\alpha_{js}|^2\right), \quad (15)$$

where σ^2 is the variance of the noise. This expression shows the interplay between the input signal to noise ratio and the SCC for the joint and factored approaches. Whereas the SCC in the factored case may not be the global optimum that can be obtained in the joint optimization, the input power is higher and consequently the output SINR is increased. Nonetheless, turning transmitters off leads to better power efficiency. It is important to note that the allowable transmit power per element may be capped (due to the available transmitter dynamic range) which may limit the gain achievable by the factored approach.

4. SIMULATION

Consider a MIMO phased array system comprising a 5-antenna uniform linear collocated array. The azimuth angle of the signal of interest is assumed to be $\phi_s = 0.3\pi$, while that of the interference azimuth, ϕ_j, is varying from 0 to $\frac{\pi}{2}$. We aim to select $k = 6$ Tx-Rx pairs for the joint problem and 2 Tx and 3 Rx elements for the factored version. The minimum SCC value is calculated first by exhaustive search for both cases and the lower bounds are computed using CVX [19]. The solution space for joint problem contains $\binom{25}{6} = 177100$, however for the factored problem this number is reduced to $\binom{5}{2} \times \binom{5}{3} = 100$. The results for both cases are depicted in Fig. 3. Firstly note that the lower bounds are tight as the curves coincide. Furthermore, the SCC obtained by joint selection is generally lower than that of the factored approach. This confirms that the joint selection achieves the global solution over all possible Tx-Rx. The factored selection, on the other hand, is suboptimal as it operates on a subspace of the possible solutions.

We now set the interference elevation $\phi_j = 0.1\pi$ and solve the selection problem for an increasing subset of antennas ranging from 1 to 25. For the factored case, the number of elements is calculated by factorizing 1 to 25 excepts the prime numbers. Where more than one factorization is possible, the one with the minimum number of Tx elements is used. As can be seen in Fig.4, the joint selection outperforms the factored version for different number of selected elements.

Finally, we compare the performance of the joint and factored approaches in terms of the output SINR. The results in Fig. 5 show that the factored approach is able to achieve a higher SINR_{out}. This is due to the increase in transmitted power per element, P_e, which counteracts the degradation in the achievable SCC value. The effect of the increase in P_e is made clear by showing the curve corresponding to a $P_e = P_T/M$ which is the value used in the joint selection. Also notice that the output SINR given by the joint selection remains comparable with that of the full array even when a significantly smaller number of pairs are used. For instance selecting 15 out of 25 Tx-Rx pairs would substantially reduce the computational cost of the STAP processing but would result in less than 0.5 dB loss with respect to the full array (13.56 dB). In the factored case, however, the increase in P_e can have a much more pronounced effect than the SCC and therefore an improvement on the full array can be obtained.

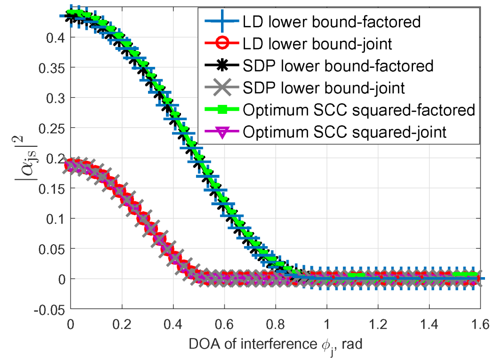

Fig. 3. Optimum SCC squared vs. lower bounds for joint Tx-Rx and factored Tx/Rx problems.

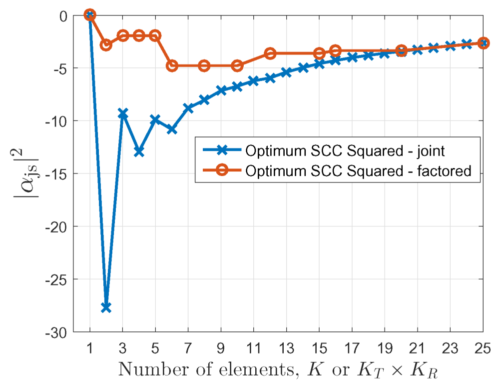

Fig. 4. Optimum SCC squared for different number of antenna subset.

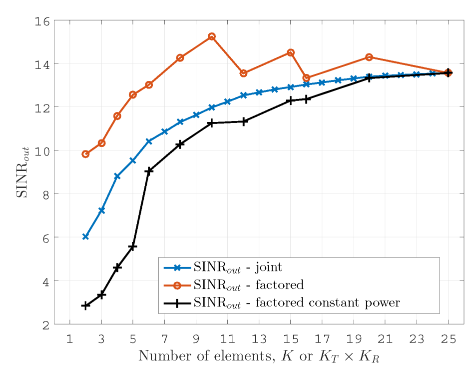

Fig. 5. $\text{SINR}_{out}$for different number of antenna subset.

5. CONCLUSION

In this paper, we proposed Tx-Rx pair selection for MIMO phased array radar in order to achieve significant hardware and computational cost savings. We formulated the joint Tx-Rx selection problem as an non-convex optimization and used relaxation methods to study the solution. Furthermore, we presented a factored version that reduces the size of the solution space, yet is able to achieve comparable or even better output signal to interference and noise ratio than the joint solution. We presented simulation results that show the effectiveness of the proposed techniques in reducing the problem dimensionality while maintaining a performance that is comparable to the full array.

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