Matched Filter Constrained MIMO Array Spatial Thinning for Interference Mitigation

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***Abstract*—The main obstacle for the implementation of multiple-input multiple-output (MIMO) phased arrays is the large number of matched filters due to the waveform diversity provided. In this paper, we propose an array thinning technique targeted at MIMO phased arrays with a limited number of matched filters in each receiver. We formulate this thinning problem as a subproblem of joint Tx-Rx selection which chooses k transmitter-receiver (Tx-Rx) pairs from the available N pairs. We then show that the solution space of the new problem is highly overlapped with the global joint selection. Hence, the matched filter constrained (MFC) selection, which operates over a far smaller space achieves the solution with less computational effort.**

Keywords*—Array thinning, MIMO radar, adaptive array beamforming, STAP, antenna selection, convex optimization.

I. Introduction

The success of multiple-input multiple-output (MIMO) communications led to the application of MIMO arrays as an effective mechanism for enhancing radar system performance [1]. This is due to the advances in orthogonal frequency division multiplexing (OFDM) and code division multiple access (CDMA), which enable transmission diversity. A MIMO phased array comprises an array of antennas, transmitting a set of noncoherent orthogonal waveforms that can be extracted at the receiver by a corresponding number of matched filters. Improved spatial diversity, parameter identifiablity, and detection performance result from the added degrees of freedom (DoFs) in MIMO phased arrays compared to the single-input multiple-output (SIMO) phased arrays [2], [3].

The advantages of the MIMO configuration are delivered at the expense of a significant increase in the problem dimensionality. For a standard SIMO system using a uniformly spaced linear array, it has been shown that the information extracted from the elements includes redundancy due you a spatial correlation lag [4]. This redundancy can be avoided either by designing a minimum-redundancy linear array (MRLA) [5], or thinning the full array by a smaller subset of elements [6]–[8]. Generally each receiving element (antenna patch) is connected to a dedicated front-end which is a great deal more expensive than the receiving element itself. The dedicated front ends add hardware cost to the computational cost that afflict large arrays. This problem is further exacerbated in large MIMO arrays where the dimensionality of the problem is given by the product of the number of receive elements with the number of transmit waveforms. Thus, the increasing use of large arrays in MIMO systems makes array thinning an important tool to reduce the redundancy as well as the computational and hardware costs.

Array thinning can be developed by adaptive spatial sampling [9] and non-uniform sparse array design [10]. Generally, the channel capacity, resolution, estimation performance, and directional detection are dependent upon the array aperture and geometry. The idea of minimum redundancy has been successfully applied to the design of physical Tx/Rx arrays to form MIMO virtual arrays with maximum contiguous aperture, i.e., minimum redundancy virtual arrays (MRVA) [11]. Furthermore, the two-level autocorrelation property of the difference sets (DSs) has been successfully exploited to maximize the virtual aperture [12].

In this work we focus on the problem of array thinning for large adaptive MIMO phased array in order to economize the system in terms of hardware and computational costs. We first introduced the idea of MIMO phased array spatial thinning for interference cancellation for spacetime-adaptive processing (STAP) in [13]. In that work we formulated the joint Tx-Rx selection problem as a non-convex optimization and used relaxation methods to study the solution. Furthermore, we presented a factored version that reduces the size of the solution space while being able to achieve comparable or even better output signal-to-interference-plus-noise-ratio (SINR) than the joint solution. Another challenge in MIMO phased arrays is the large number of matched filters needed for the full array. As noted the dimensionality of the signal space in a MIMO system is given by the number of matched filters multiplied by the number of orthogonal waveforms, which can be quite large. We propose to apply array thinning to the reduce the number of matched filters. In particular, we propose MIMO phased array spatial thinning for interference mitigation in a STAP framework when there is limited number of matched filter in each receiver.

The paper is organised as follows. The idea of MIMO array spatial thinning is reviewed in section II. Then the matched filter constrained (MFC) version is elaborated in section III and the lower bounds and discussion is presented in section IV. Finally some conclusions are drawn in section V.

II. MIMO ARRAY THINNING BY JOINT TX-RX SELECTION

The performance of MIMO phased arrays can be characterized by a larger virtual array. The positions in the new virtual array are obtained as the combination of the location of the transmitters by the receivers. Let the locations of the transmitters $\mathbf{P}_{\mathrm{T}}$ and receivers $\mathbf{P}_{\mathrm{R}}$ be defined as

$$\mathbf{P}_{\mathrm{T}} = \begin{bmatrix} x_{\mathrm{T},1} & y_{\mathrm{T},1} \\ x_{\mathrm{T},2} & y_{\mathrm{T},2} \\ \vdots & \vdots \\ x_{\mathrm{T},M} & y_{\mathrm{T},M} \end{bmatrix}, \quad \mathbf{P}_{\mathrm{R}} = \begin{bmatrix} x_{\mathrm{R},1} & y_{\mathrm{R},1} \\ x_{\mathrm{R},2} & y_{\mathrm{R},2} \\ \vdots & \vdots \\ x_{\mathrm{R},N} & y_{\mathrm{R},N} \end{bmatrix}. \tag{1}$$

Then, the output convolution can be expressed in terms of the MN virtual array elements as follows

$$\mathbf{P}_{\mathrm{V}} = \mathbf{P}_{\mathrm{R}} \otimes \mathbf{1}_M + \mathbf{1}_N \otimes \mathbf{P}_{\mathrm{T}}, \tag{2}$$

where $\otimes$ is the Kronecker product and $\mathbf{1}_N$ is a length-N vector with elements equal to 1.

The spatial steering vectors of the desired signal and an interference coming from (ϕ_s, θ_s) and (ϕ_j, θ_j) respectively are given by

$$\mathbf{v}_s = e^{j\frac{2\pi}{\lambda}\mathbf{P}_{\mathrm{V}}\mathbf{u}_s}, \mathbf{v}_j = e^{j\frac{2\pi}{\lambda}\mathbf{P}_{\mathrm{V}}\mathbf{u}_j}, \tag{3}$$

where

$$\mathbf{u}_i = [\sin\theta_i \cos\phi_i \quad \sin\theta_i \sin\phi_i]^T. \tag{4}$$

The spatial separability of the desired signal and interference can be represented by the angle between the signal and interference subspaces. It is measured by the spatial correlation coefficient as follows

$$\alpha_{js} = \frac{\mathbf{v}_j^H \mathbf{v}_s}{\|\mathbf{v}_j\| \, \|\mathbf{v}_s\|} = \frac{\mathbf{v}_j^H \mathbf{v}_s}{\sqrt{\mathbf{v}_j^H \mathbf{v}_j}\sqrt{\mathbf{v}_s^H \mathbf{v}_s}} = \frac{\mathbf{v}_j^H \mathbf{v}_s}{MN}. \tag{5}$$

Specifically, the SCC gives the cosine of the angle between the two subspaces. Therefore, the SCC is equal to 0 when the signal and interference are mutually orthogonal. In general, a smaller SCC relates to a larger separation between the two subspaces.

Now suppose that we want to select a subset k of matched filters out of the MN available filters. We introduce a length MN selection vector $\mathbf{c}$ comprising entries that are 1 or 0, with 1 indicating that the corresponding MF is selected and 0 that it is not. The SCC can then be expressed as [8]

$$|\alpha_{js}|^2 = \frac{\mathbf{c}^T \mathbf{W}_r \mathbf{c}}{k^2}, \tag{6}$$

where $\mathbf{W}_r$ is

$$\mathbf{W}_r = \mathrm{real}(\mathbf{v}_{js}\mathbf{v}_{js}^H), \tag{7}$$

and

$$\mathbf{v}_{js} = \mathbf{v}_s \odot \mathbf{v}_j^T. \tag{8}$$

For the vector $\mathbf{v}$, we use $\mathbf{v}^H$ and $\mathbf{v}^T$ to denote the complex Hermitian transpose and transpose, respectively. The notation $\odot$ represents Hadamard product and $\mathrm{real}(\cdot)$ denotes the real part of a complex matrix or vector.

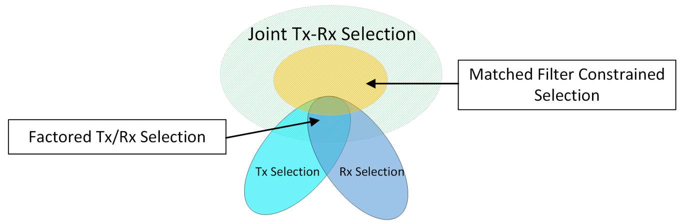


Fig. 1. The solution space in MIMO spatial array thinning.

Using the above formulation, a minimization problem can then be constructed with the aim of enhancing the separation between the signal and interference subspaces. This is equivalent to minimising the SCC and the problem is expressed as

$$\begin{aligned} &\min_{\mathbf{c}} \ |\alpha_{js}|^2 \\ &\text{s.t. } c_i(c_i - 1) = 0 \quad i = 1...MN, \\ &\text{and } \mathbf{c}^T\mathbf{c} = k. \end{aligned} \tag{9}$$

We now rewrite the selection vector as the following selection matrix

$$\mathbf{C} = \begin{matrix} & \mathrm{Tx}_1 & \mathrm{Tx}_2 & \cdots & \mathrm{Tx}_M \\ \mathrm{Rx}_1 & c_{1,1} & c_{1,2} & \cdots & c_{1,M} \\ \mathrm{Rx}_2 & c_{2,1} & c_{2,2} & \cdots & c_{2,M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathrm{Rx}_N & c_{N,1} & c_{N,2} & \cdots & c_{N,M} \end{matrix}, \tag{10}$$

such that,

$$\mathbf{c} = \mathrm{vec}\{\mathbf{C}\}, \quad NM \times 1.$$

Therefore, $c_{i,j}$ is the element that indicates whether the j-th matched filter (which extracts the j-th waveform) in the i-th receiver is selected. In the general joint selection case, we choose the matrix entries without any additional restriction on the structure.

This problem can be interpreted as the selection of a subset of elements from the virtual array (see Fig. 1). However, the feasible solution space for this optimization has some important subspaces. The joint selection optimization problem puts a constraint only on the number of output matched filters. Since any subset of filters is a possible solution, the entire set of transmitters must necessarily transmit their waveforms. If the matched filters corresponding to a particular transmit element are not used, keeping the transmitting element active leads to wasted transmit power. This situation can be avoided by factorizing the selection problem into a transmit and receive sub-problems. Suppose that we select k_{T} out of M transmitting elements and k_{R} out of the available N receive elements. The overall selection matrix becomes $\mathbf{C} = \mathbf{c}_{\mathrm{T}} \times \mathbf{c}_{\mathrm{R}}^T$, where $\mathbf{c}_{\mathrm{R}}$ and $\mathbf{c}_{\mathrm{R}}$ are the selection vectors for transmit and receive sides receptively. To factorize the problem we write the selection vector as $\mathbf{c} = \mathbf{c}_{\mathrm{R}} \otimes \mathbf{c}_{\mathrm{T}}$. In [13] we showed that this formulation leads to a factored problem with transmitter and receiver factors. The feasible subspace for the factored problem is the intersection of the optimum Tx and Rx feasible subspaces as shown in Fig. 1.

The factored formulation enables us to manage the transmitter power and as a result the SNR. Moreover, the number of receivers is decreased which leads to a great hardware reduction. However, a specific receiver should be active with all of the corresponding matched filters. Additionally, as shown in our previous work the number of transmitters and receivers is reduced at the expense of diminished spatial diversity. To maintain the spatial diversity that is offered by MIMO arrays while at the same time reducing the hardware and computation overheads, the selection should be applied directly to the number of matched filters in each receiver. In this way, the spatial diversity can be preserved yet the system dimension can be significantly decreased.

III. Matched filter constrained MIMO array thinning

Placing a limit on the number of matched filters in each receiver requires control over the sum of the rows in the selection matrix $\mathbf{C}$. Suppose we have only k_M available matched filters in each receiver, then we rewrite the squared SCC as

$$|\alpha_{js}|^2 = \frac{\text{vec}\{\mathbf{C}\}^T \mathbf{W}_r \text{vec}\{\mathbf{C}\}}{k_{\text{M}}^N}. \quad (11)$$

The restriction on the number of active elements in each row to k_M is expressed in the optimization problem as the equality constraint

$$\mathbf{C}\ \mathbf{1}_M = \mathbf{k}_M, \quad N \times 1. \quad (12)$$

Thus, a matched filter thinned MIMO phased array design that is aimed at making the signal and interference subspaces as orthogonal as possible is defined as follows

$$\begin{aligned} &\min_{\mathbf{C}} |\alpha_{js}|^2 \\ &\text{s.t. } c_{i,j}(c_{i,j}-1) = 0 \qquad i = 1...M, \quad j = 1...N \\ &\quad \mathbf{C}\ \mathbf{1}_M = \mathbf{k}_M. \end{aligned} \quad (13)$$

The binary constraints make the optimization problem a two-way partitioning and hence a non-convex problem. Thus, we first verify whether the problem is bounded. To this end, we begin by reformulating the optimisation using the Lagrangian relaxation as

$$\begin{aligned} L(\mathbf{c}, \boldsymbol{\mu}, \boldsymbol{\nu}) &= \text{vec}\{\mathbf{C}\}^T \mathbf{W}_r \text{vec}\{\mathbf{C}\} \\ &+ \text{vec}\{\mathbf{C}\}^T \text{diag}(\boldsymbol{\mu}) \text{vec}\{\mathbf{C}\} - \boldsymbol{\mu}^T \text{vec}\{\mathbf{C}\} \\ &+ \boldsymbol{\nu}^T (\mathbf{C} \times \mathbf{1}_M) - \boldsymbol{\nu}^T \mathbf{k}_M \\ &= \text{vec}\{\mathbf{C}\}^T (\mathbf{W}_r + \text{diag}(\boldsymbol{\mu})) \text{vec}\{\mathbf{C}\} \\ &- \boldsymbol{\mu}^T \text{vec}\{\mathbf{C}\} + (\mathbf{1}_M \otimes \boldsymbol{\nu})^T \text{vec}\{\mathbf{C}\} - \boldsymbol{\nu}^T \mathbf{k}_M \\ &= \text{vec}\{\mathbf{C}\}^T (\mathbf{W}_r + \text{diag}(\boldsymbol{\mu})) \text{vec}\{\mathbf{C}\} \\ &+ ((\mathbf{1}_M \otimes \boldsymbol{\nu}) - \boldsymbol{\mu})^T \text{vec}\{\mathbf{C}\} - \boldsymbol{\nu}^T \mathbf{k}_M, \end{aligned} \quad (14)$$

where $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ are $MN \times 1$ and $N \times 1$ Lagrange multiplier vectors. The Lagrangian dual function for the quadratic form of (14) is written as follows

$$g(\boldsymbol{\mu}, \boldsymbol{\nu}) = \inf_{\mathbf{c}} \{L(\mathbf{c}, \boldsymbol{\mu}, \boldsymbol{\nu})\}$$
$$\begin{cases} -\frac{1}{4}\left((\mathbf{1}_M \otimes \boldsymbol{\nu}) - \boldsymbol{\mu}\right)^T & \left(\frac{1}{k_M^{2N}} \mathbf{W}_r + \text{diag}(\boldsymbol{\mu})\right)^{-1} \times \\ & ((\mathbf{1}_M \otimes \boldsymbol{\nu}) - \boldsymbol{\mu}) - \boldsymbol{\nu}^T \mathbf{k}_M \\ & \text{if } \frac{1}{k_M^{2N}} \mathbf{W}_r + \text{diag}(\boldsymbol{\mu}) \succeq 0 \\ -\infty \quad \text{otherwise.} \end{cases} \quad (15)$$

Using the Schur complement condition for positive semidefiniteness of a block matrix, the concave function listed in (15) is formulated as the maximization problem,

$$\begin{aligned} &\max_{\boldsymbol{\mu}, \boldsymbol{\nu}} g \\ &\text{s.t} \begin{bmatrix} \frac{1}{k_M^{2N}} \mathbf{W}_r + \text{diag}(\boldsymbol{\mu}) & -\frac{1}{2}\left((\mathbf{1}_M \otimes \boldsymbol{\nu}) - \boldsymbol{\mu}\right) \\ -\frac{1}{2}\left((\mathbf{1}_M \otimes \boldsymbol{\nu}) - \boldsymbol{\mu}\right)^T & -\boldsymbol{\nu}^T \mathbf{k}_M - g \end{bmatrix} \succeq 0. \end{aligned} \quad (16)$$

The above convex optimization problem is a semidefinite program that can solved using CVX [14]. Based on the assumed duality, the obtained maximum value forms a lower bound for the problem in (13).

IV. Simulation

For the simulation task, we use a uniform linear collocated MIMO phased array containing 5 elements, in which the azimuth angle of the signal of interest is fixed at $\phi_s = 0.3\pi$ and the azimuth of the received interference, ϕ_j varies from 0 to $\frac{\pi}{2}$. In this scenario we define the problem such that we select first $k = 10$ matched filters in the global space without any spatial restriction. In the second case, we choose $k_M = 2$ matched filter in each receiver leading to $k = 10$ elements in total. We calculate the Lagrange dual lower bound for both cases. The lower bound for the first case is calculated based on [13], while for the MFC case the formulation mentioned in (16) is employed. Also, the optimum solution is found by an exhaustive search for both schemes. Four curves including the lower bounds and optimum SCC squared values are shown in 2. The results demonstrate that the bounds are tight. Furthermore, referring to the feasible solution space depicted in Fig. 1, we conclude that the solution set of the matched filters constrained case is a subset of the solution set of the global optimization problem. An example solution that compares the result of the joint and MFC selection approaches is depicted in Fig. 3, where Fig. 3(a) displayes the unconstrained selection of 10 matched filters from the matrix regardless to the receiver (row), whereas 3(b) gives the result of selecting two matched filters per receiver.

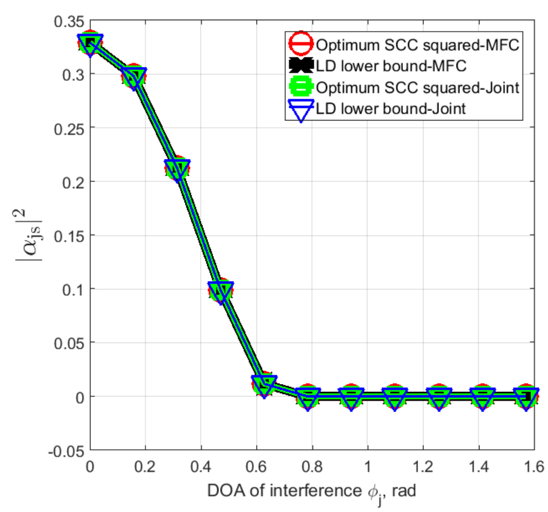
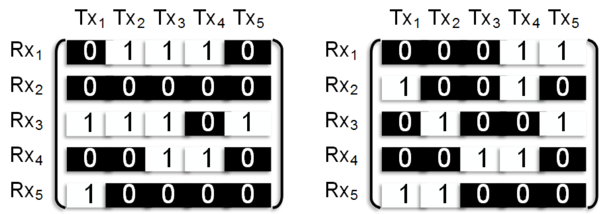


Fig. 2. Optimum SCC squared value vs. lower bounds for joint Tx-Rx and MFC selection.



(a) (b)

Fig. 3. Typical optimum selection matrix for the simulated scenario (a): Joint selection. (b): MFC selection.

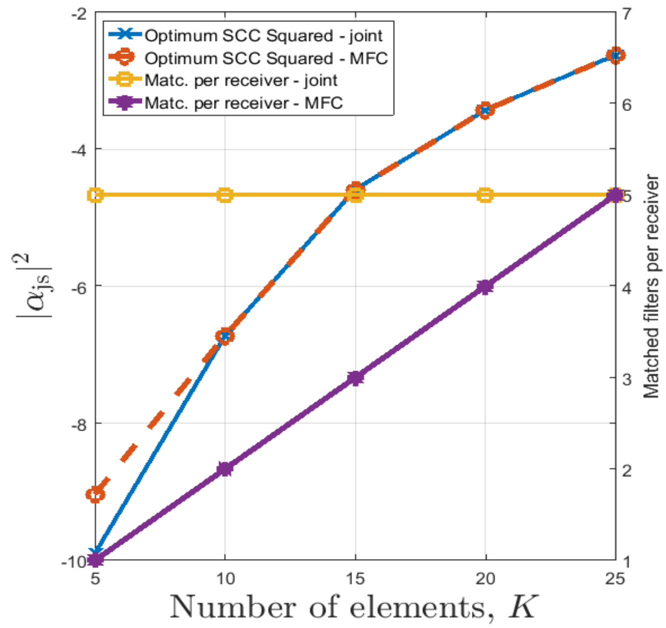


Fig. 4. Optimum SCC squared value for different numbers of antenna subset.

In the second simulation, we evaluate the selection strategies for a fixed signal and interference scenario but for different numbers of selected elements ranging from $k_M = 1$ matched filter per receiver ($k = 5$ total matched filters selected) to the full matched filter set in each receiver (that is $k_M = 5$ giving a total of $k = 25$ selected elements). The azimuth angle of the interference is fixed at $\phi_j = \frac{\pi}{10}$. The results observed in Fig. 2 are confirmed in this simulation task as well as shown in Fig. 4. Apart from the case where only one matched filter in each receiver is chosen, the minimum SCCs obtained by both strategies coincide. Also, the number of required matched filters in each receiver is depicted in Fig. 4. To obtain the minimum SCC in the joint selection all matched filters should be always active, which implies that in the most general case, only computational saving is achieved. However, by restricting the number of matched filter in each receiver the same resolution is attainable along with a significant hardware saving. Additionally, the feasible space is much smaller in the MFC case. For instance in order to choose $k = 10$ elements out of 25 filters we should search for the solution within a space containing $\binom{25}{10} = 3268760$, but this search space is reduced to a substantially smaller space containing $\binom{5}{2}^5 = 100000$. The smaller space is approximately only 3% of the global feasible space.

To study the effect of the array thinning output on the post STAP algorithm we calculate the SINR_{out}

$$SINR_{\text{out}} = \alpha k \frac{P_{\text{T}}}{\sigma^2}\left(1 - |\alpha_{js}|^2\right), \tag{17}$$

Where α represents the channel gain (including target cross-section), P_{T} is the available transmit power, and σ^2 the variance of the noise. The normalized SINR_{out} is given in Fig. 5. We observe that, for instance, selecting 15 elements out of 25 either jointly or 3 matched filter per receiver, SINR_{out} leads to a SINR degradation of only by 1.2 dB. This may be a small, acceptable value when considering that the corresponding hardware and computation cost that is achieved by MFC selection is quite significant.

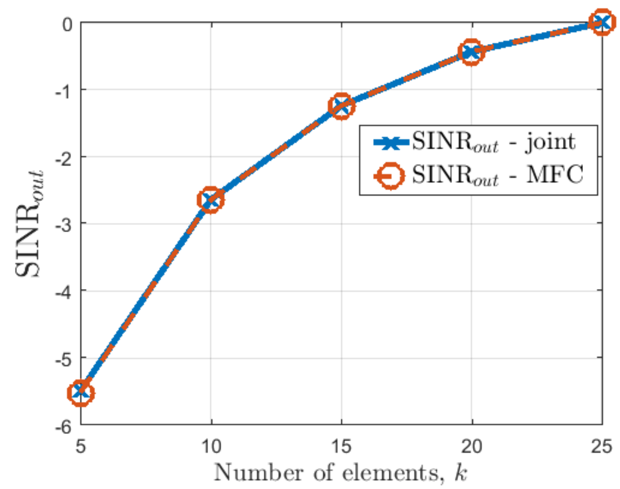


Fig. 5. $\text{SINR}_{out}$for different numbers of antenna subset.

V. Conclusion

In this paper we proposed a MIMO phased array spatial thinning scheme that successfully thins the adaptive matched filter banks in each receiver. We expressed the matched filter constrained (MFC) selection problem as a reformulation of the global joint spatial MIMO thinning aimed at interference mitigation. We then proceeded to relax the non-convex MFC selection using the Lagrange Dual Relaxation. This allowed the lower bound was to be calculated based on the achieved dual concave function. We showed through the attained lower bound demonstrated that the solution space of the MFC selection is a subset of the joint global selection case. Hence, the computation and hardware savings obtained by joint selection can be considerably enhanced by the proposed method.

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