Price Discrimination for Energy Trading in Smart Grid: A Game Theoretic Approach

Wayes Tushar, Member, IEEE, Chau Yuen, Senior Member, IEEE, David B. Smith, Member, IEEE, and H. Vincent Poor, Fellow, IEEE

Abstract-Pricing schemes are an important smart grid feature to affect typical energy usage behavior of energy users (EUs). However, most existing schemes use the assumption that a buyer pays the same price per unit of energy to all suppliers at any particular time when energy is bought. By contrast, here a discriminate pricing technique using game theory is studied. A cake cutting game is investigated, in which participating EUs in a smart community decide on the price per unit of energy to charge a shared facility controller (SFC) in order to sell surplus energy. The focus is to study fairness criteria to maximize sum benefits to EUs and ensure an envy-free energy trading market. A benefit function is designed that leverages generation of discriminate pricing by each EU, according to the amount of surplus energy that an EU trades with the SFC and the EU's sensitivity to price. It is shown that the game possesses a socially optimal, and hence also Pareto optimal, solution. Further, an algorithm that can be implemented by each EU in a distributed manner to reach the optimal solution is proposed. Numerical case studies are given that demonstrate beneficial properties of the scheme.

Index Terms—Smart grid, cake cutting game, shared facility, discriminate pricing, social optimality, Pareto optimality.

I. INTRODUCTION

O NE OF the main stimuli behind adopting energy management in smart grid is the use of different pricing schemes, in which an energy entity changes the price of per unit electricity according to the generation and demand so as to motivate users to modify their attitudes toward electricity consumption and supply [1]–[3]. Particularly, with the advancement of distributed energy resources (DERs), different pricing

Manuscript received June 14, 2015; revised August 18, 2015 and November 16, 2015; accepted December 9, 2015. Date of publication January 8, 2016; date of current version June 19, 2017. This work was supported in part by the Singapore University of Technology and Design through the Energy Innovation Research Program Singapore under Grant NRF2012EWT-EIRP002-045, and in part by the U.S. National Science Foundation under Grant ECCS-1549881. The work of D. B. Smith was supported in part by the National ICT Australia, in part by the Australian Government through the Department of Communications, and in part by the Australian Research Council through the Information and Communications Technology Centre of Excellence Program. Paper no. TSG-00677-2015.

W. Tushar and C. Yuen are with the Singapore University of Technology and Design, Singapore 487372 (e-mail: wayes_tushar@sutd.edu.sg; yuenchau@sutd.edu.sg).

D. B. Smith is with National ICT Australia, Australian Technology Park, Eveleigh, NSW 2015, Australia (e-mail: david.smith@nicta.com.au).

H. V. Poor is with the School of Engineering and Applied Science, Princeton University, Princeton, NJ 08544 USA (e-mail: poor@princeton.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSG.2015.2508443

techniques can assist the grid or other energy entities, such as shared facility controllers $(SFCs)^1$ to operate reliably and efficiently by obtaining some energy supply from the energy users (EUs) [4].

Over the past few years there has been significant interest in devising pricing schemes for energy management in smart grid. These schemes can be classified into three general categories: time-of-use pricing; day-ahead pricing; and real-time pricing [5]. Time-of-use pricing has three different pricing rates: peak, off-peak and shoulder rates based on the time of electricity use by the EUs [6]. Day-ahead pricing is determined by matching offers from generators to bids from EUs in order to develop a classic supply and demand equilibrium price at an hourly interval [7]. Finally, real-time pricing [5] refers to tariffed retail charges for delivering electric power and energy that vary over hour-to-hour, and are determined through an approved methodology from wholesale market prices. Other pricing schemes that have been discussed in the literature include critical peak pricing, extreme day pricing, and extreme day peak pricing [5]. Discussion of various pricing schemes for energy management in smart grid can be found in [8]-[15] and the references therein. Nevertheless, an important similarity in most of these pricing schemes is that all of the EUs decide on the same selling price per unit of energy at a particular time.

With the increase in government subsidies for encouraging the use of renewables, more EUs with DERs are expected to be available in the future [16]–[21]. This will subsequently lead to a better completion of purchasing targets for SFCs in order to maintain electricity for shared facilities in a community [22]. This is due to the fact that the opportunity of an SFC to trade electricity with EUs can greatly reduce its dependence on the grid, and consequently decrease the cost of energy purchase. However, not all EUs would be interested in trading their surplus energy if the benefit from the SFC is not attractive [4]. In particular, as we will see shortly, this can happen to EUs with limited energy surplus and/or with higher sensitivity to price whose respective return could be very small. Nevertheless, as shown in [4], one possible way to address this problem is that these EUs can sell their energy at a relatively higher price per unit of energy, within a reasonable margin, compared to EUs with very large DERs (and/or, with lower sensitivity to the choice of price) without affecting their revenue significantly.

1949-3053 © 2016 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission. See http://www.ieee.org/publications_standards/publications/rights/index.html for more information.

¹An SFC, as we will see in the next section, is an entity that is responsible for managing energy of different shared facilities of a smart community.

It is natural to think that the benefit to the end-user will increase if the price for selling each unit of energy increases. However, we note that in an energy market with a large number of sellers the buyer has many choices to buy electricity. Hence, a significantly higher price per unit of energy from a seller can motivate the buyer to switch its buying from that expensive option to a seller who is selling at a comparatively cheaper rate [23]. Thus, even with a higher selling price per unit of energy, the net benefit to a user may decrease significantly if the amount of energy that it can sell to the buyers, i.e., the SFC in this case, becomes very small. This type of phenomenon has occurred recently in the global oil market [24]. This can further be illustrated by a toy example as follows.

Consider the numerical example given in Table I where EU1 and EU2 sell their surplus energy to the SFC to meet the SFC's 40 kWh energy requirement. It is considered that EU1 and EU2 have DERs with capacity of 50 kWh and 10 kWh respectively (and EU1 is significantly larger than EU2) in terms of available energy to supply). In case 1, EU1 and EU2 sell 35 and 5 kWh of energy to the SFC at a price of 20 cents/kWh. Hence, the revenues of EU1 and EU2 are 700 and 100 cents respectively and the total cost to the SFC is 800 cents. In case 2, EU1 and EU2 choose their prices differently and sell their surplus energy at rates of 18 cents/kWh and 22 cents/kWh respectively. Now, due to the change of price in case 2, if EU1 reduces its selling amount to 32 (since the price is reduced) and EU2 increases its surplus amount to 8 kWh (as the current price is high) the resulting revenue changes to 576 and 176 cents respectively for EU1 and EU2 whereas the total cost to the SFC reduces to 752 cents. Thus, according to this example, it can be argued that discriminate pricing is considerably beneficial to EUs with small energy (revenue increment is 76%) at the expense of relatively lower revenue degradation (e.g., 17% in case of EU1) from EUs with larger energy capacity. It further reduces the cost to the SFC by 6%.

However, one main challenge for such price discrimination among different EUs, which is not discussed in [4] and yet needs to be explored, is the maintenance of fairness of price distribution between different EUs to enable such schemes to be sustained in electricity trading markets. For example, if the EUs are not happy with the price per unit of energy that they use to sell their surplus energy, or if they envy each other for the adopted discrimination, energy markets that practice such discriminate pricing schemes would eventually diminish. Hence, there is a need for solutions that can maintain the price disparity between EUs in a fair manner, whereby considering their available surplus energy trading environment with a view to obtain a socially optimal² energy management solution is ensured.

To this end, this paper complements the existing pricing schemes in the literature by studying the fairness of selecting different prices for different EUs in smart grid. However,

TABLE I NUMERICAL EXAMPLE OF A DISCRIMINATE PRICING SCHEME WHERE AN SFC REQUIRES 40 kWh OF ENERGY FROM TWO EUS AND THE SFC'S TOTAL PRICE PER UNIT OF ENERGY TO PAY TO THE EUS IS 40 CENTS/kWh

	Case 1	Case 2
Payment to EU1 (cents/kWh)	20	18
Payment to EU2 (cents/kWh)	20	22
Energy supplied by EU1 (kWh)	35	32
Energy supplied by EU2 (kWh)	5	8
Revenue of EU 1 (cents)	700	576 (-17%)
Revenue of EU 2 (cents)	100	176 (+76%)
Cost to the SFC (cents)	800	752 (-6%)

unlike [4], where a two-stage Stackelberg game is studied, we take a different approach in this paper. Particularly, we explore a cake-cutting game (CCG) for selecting discriminate prices for different users. In the proposed CCG, the EUs with smaller available energy can decide on a higher unit price, and the price is also adaptive to the sensitivity of EUs to the choice of price. A suitable benefit function is chosen for each of the EUs that enables the generation of discriminate pricing so as to achieve a *socially optimal* solution of the game. Thus, also, Pareto optimality is directly implied by this socially optimal solution, and hence an envy-free energy trading market is established. We propose an algorithm that can be adopted by each EU in a distributed manner to decide on the price vector by communicating with the SFC, and the convergence of the algorithm to the optimal solution is demonstrated. Finally, we present some numerical case studies to show the beneficial properties of the proposed discriminate pricing scheme.

We stress that the current grid system does not allow such discriminate pricing among EUs. Nonetheless, the idea of price discrimination is not new and has been extensively used in economic theory. For instance, the effect of price discrimination on social welfare is first investigated in 1933 [26], which is further extended with new results in [27] and [28]. In recent years, the authors in [29] study the airport congestion pricing technique when the airline carriers discriminate with respect to price. In [30], the authors use a new panel of data on buyer-supplier transfers and build a structural model to empirically analyze bargaining and price discrimination in a medical device market. The study of intertemporal price discrimination between consumers who can store goods for future consumption needs is presented in [31], and the effects of third-degree price discrimination on aggregate consumers surplus is considered in [32]. Further, a framework for flexible use of cloud resources through profit maximization and price discrimination is studied in [33]. In this context, we also envision discriminate pricing as a further addition to real-time pricing schemes in future smart grid. Such a scheme is particularly suitable for the energy trading market when the SFC may want to reduce its dependence on a single dominant user. For example, in the toy example the SFC may rely heavily on User 1 who has a large surplus for the same price model. However, by giving more incentive to User 2, the SFC managed to reduce its dependence on User 1 by buying less energy from it. Please note

 $^{^{2}}$ A socially optimal solution maximizes the sum of benefits to all EUs in the smart grid network [25].

that this could happen in a real-world scenario in which a buyer pays a small supplier a relatively higher price in order to help the small supplier grow, and at the same time to reduce the dependence on a single big supplier. Such trading will prevent the possibility of the big supplier growing too big and creating a monopoly, which could lead to a serious problem in the long run. Please note that examples of such differentiation can also be found in current standard Feed-in-Tariff (FIT) schemes [34].

The remainder of the paper is organized as follows. The system model is described in Section II and the problem of price discrimination is studied as a CCG in Section III. We provide numerical case studies to show the beneficial properties of the proposed scheme in Section IV. Finally, we offer some concluding remarks in Section V.

II. SYSTEM DESCRIPTION

Consider a smart grid system consisting of N EUs, where $N = |\mathcal{N}|$, an SFC and a main grid. Each EU $n \in \mathcal{N}$ can be considered as a single user or a group of users connected via an aggregator that acts as a single entity. Each EU is equipped with DERs such as solar panels and wind turbines and does not have any storage facility.³ Hence, EU n needs to sell its surplus energy, if there is any, either to the SFC or to the main grid after meeting its essential demand in order to make some extra revenue. Due to the fact that the buying price $p_{g,buy}$ of a main grid is considerably lower than pricing within a facility [36], it is reasonable to assume that the EUs would be more keen to sell their surplus to the SFC instead of to the grid.⁴ Alternatively, if the payment from the SFC is not sufficiently attractive to any EU, the EU may schedule its equipment for extra consumption or may choose to sell to the grid instead of selling to the SFC. The SFC, on the other hand, controls the electricity consumption of equipment and machines that are shared and used by the EUs on a regular basis, and does not have any energy generation capacity. Therefore, the SFC relies on EUs and the grid for its required electricity. Essentially, the SFC is interested in buying as much energy as possible from the EUs as the buying price per unit of energy from the main grid is significantly higher. All the EUs, the main grid and the SFC are connected to each other through power and communication lines [1]. A schematic representation of the considered system model is shown in Fig. 1.

To this end, let us assume that at a particular time during a day each EU $n \in \mathcal{N}$ has an energy surplus of e_n that it wants to sell either to the SFC or to the grid with a view to making extra revenue. Each EU n charges the SFC a price p_n per unit of energy for selling its e_n . p_n , $\forall n$ do not need to be equal to each other and can be varied according to the amount of surplus at each EU and the EU's sensitivity $\alpha_n > 0$ to the choice of price. The SFC, on the other hand, wants to buy this energy e_n from each EU n in order to meet the demands



Fig. 1. Demonstration of the elements of the considered system model and the direction of the flow of power and information within the system.

of the shared facilities of the community. We assume that the SFC has a budget constraint C, and hence, the sum of what the SFC needs to pay to all EUs needs to satisfy

$$\sum_{n} e_{n} p_{n} \le C. \tag{1}$$

Please note that such a budget is necessary to prevent the EUs from increasing their selling price per unit of energy considerably and thus maintaining market competitiveness, which may arise due to allowing EUs to decide on the selling price through interacting with the SFC.⁵ This budget also enables us to decouple the SFC's decision making process of buying energy from the EUs (which is the main focus of this work) from the problem of the SFC's buying energy from the grid. However, the budget, which facilitates price discrimination in the proposed scheme, needs to be chosen such that it is always lower than the total price that the SFC needs to spend buying the same amount of energy from the grid and thus always benefits the SFC in terms of reducing cost. This is due to the fact that if the total money that the SFC pays to the EUs becomes equal to (or greater than) the amount that it needs to pay the grid, the SFC will not be encouraged to buy energy from the EUs as the SFC can buy all its required energy from the grid independently. Nevertheless, a suitable value for a budget Cmay depends on many other factors, e.g., how willing a user is to sell its surplus to the SFC, which requires human behavioral models to obtain reasonable estimates. For example, as a rational entity, each EU n would be interested in charging the SFC the largest price per unit of energy possible for selling its surplus. However, on the one hand, a very high price may discourage the SFC from trading any energy with the EU and rather motivate the SFC to buy its energy from the grid. On the other hand, if the selling price of an EU is too small, this may compel the EU to withdraw its surplus from the energy market as the expected revenue from energy trading with the SFC would be significantly lower. Nonetheless, in this paper

³An example of such a system is a grid-tie solar system without a storage device [35].

⁴For example, in the state of Queensland in Australia, the selling price of electricity is 17.898 cents/kWh during off peak hours (which is almost double during peak hours) [37], whereas the buying price of electricity is 6.348 cents/kWh under Queensland's Feed-in Tariff scheme [38].

⁵As we will see later, EUs and the SFC interact with each other in the proposed scheme to decide on the price vector.

we consider a general setting, i.e., a budget C for the SFC when it buys energy from the EUs.

Now, the choice of price p_n by an EU n is also restricted by its sensitivity to the choice of price. For example, as discussed in Table I, a smaller price may not affect an EU with a very large surplus (i.e., lower sensitivity), but it can significantly alter the total revenue of an EU with lower available energy (higher sensitivity). In this context, it is considered that the price p_n per unit of energy that an EU *n* asks from the SFC depends not only on the available energy e_n to the EU but also on the EU's sensitivity α_n to the choice of price, which is chosen motivated by the use of preference/reluctance parameters in [22], [23], and [39]–[41]. α_n captures the sensitivity of each EU's benefit from changing the per unit price and thus is used to quantify the different types of players. For example, if one user has a large amount of surplus, it may want to sell the energy at a relatively cheaper price compared to a player who has a small surplus to sell. This is due to the fact that players with larger amount of surplus energy might be more interested in selling all the energy for a higher gain and thus will be flexible in reducing their price for selling more energy. Hence, a relatively lower price may not affect their revenues significantly [15]. On the other hand, a user with a small energy surplus will not be more keen to sell energy unless the price per unit of energy is considerably higher as otherwise the expected return will be very small. Hence, the evaluation of a change in price per unit of energy as well as the willingness to increase the price may not be same to both the players. We capture this aspect through a parameter α_n , which is multiplied by the price per unit of energy (i.e., to capture the fact that a similar price may be interpreted differently by different EUs).

Also, as a rational entity each EU *n* wants to increase the price p_n per unit of energy that it charges the SFC as much as possible. However, the maximum price P_n chosen by each *n* needs to be such that it does not exceed the grid's selling price p_g per unit of energy. For example, if P_n is greater than grid's selling price p_g , clearly the SFC will not buy any energy from the EU. To this end, each EU *n* may want to increase p_n to a maximum value of P_n per unit of energy for selling its surplus and the choice of price p_n is determined by 1) the surplus e_n available to EU *n*, 2) the EU's sensitivity α_n to the choice of price p_n , and 3) finally, the budget available to the SFC such that (1) is satisfied.

Now, to determine the energy trading parameter $\mathbf{p} = [p_1, p_2, \dots, p_n, \dots, p_N]$, each EU *n* interacts with the SFC to decide on the price p_n per unit of energy that it wants to charge the SFC for selling its energy e_n with a view to maximizing its benefit. To capture the benefit to each EU *n*, we propose a benefit function⁶ X_n . In standard game theoretic research, e.g., [42], the benefit function is an input to the game, whose outcome needs to be a real number, and illustrates the change in benefits corresponding to the change of a player's choice of action or environmental parameters. Note that benefit functions can be a combination of parameters with different dimensions and units to capture the effects of the change of parameters by

the players. For instance, in [43], the authors consider a utility function that is a combination of transmission rate and the cost of transmission in order to show how different choices of price and transmission rate can affect the benefits to the respective player. In [41], the authors use a welfare function for their game, in which the welfare function is a combination of total cost of energy trading and the square of the amount of energy traded by the player. Further, a non-cooperative game is proposed in [44], in which a utility function combines the cost of energy and the quantity of energy to be sold.

Now, we propose a benefit function X_n , which is based on a linearly decreasing marginal benefit,

$$\tilde{X}_n = P_n - \alpha_n p_n - e_n, \tag{2}$$

contingent on the following assumptions:

- 1) Each EU *n* is a rational entity and wants to choose a price p_n per unit of energy as close to the maximum possible price P_n , e.g., equal to the grid's selling price, as possible.
- 2) EU *n* is sensitive to its choice of price per unit of energy through the parameter α_n , and thus the choice of p_n is restricted by the choice of α_n .
- An EU with large surplus of energy has relatively lower marginal benefit compared to EUs with lower surplus for the same choice of price [4].

Note that a utility function with decreasing marginal benefit is shown in [45] to be appropriate for energy users. Then, the same property is also used to design utility functions in [15], [23], [40], and [41], where the players participate in games for making decisions on energy trading parameters including price and energy. To this end, the chosen utility function in (3) also possesses the property of declining marginal benefit and is close to the utility functions used in [15], [23], and [41]. Please note that the authors modeled a two-level game in [41] and Stackelberg games in [15] and [23] for designing energy management for smart grid. Therefore, a similar property is used to model the utility function of the proposed game.

To that end, the net benefit X_n that an EU *n* can attain from selling its surplus e_n to the SFC at a price p_n can be defined as

$$X_n(p_n) = \int_0^{p_n} \tilde{X}_n dq_n$$

= $P_n p_n - \frac{\alpha_n}{2} {p_n}^2 - e_n p_n.$ (3)

The benefit function in (3) is a quadratic function of p_n , which leverages the generation of discriminate pricing [4] between different EUs of the system. As can be seen in (3), the proposed benefit function possesses the following properties:

- 1) The benefit function is a concave function of p_n , i.e., $\frac{\delta^2 X_n}{\delta p_n^2} < 0$. Hence, the benefit of an EU may decrease for an excessively high p_n . This models the fact that if the price is very high, the SFC may restrain from buying energy from the EU, which will eventually reduce its expected benefit from energy trading.
- 2) The benefit function X_n is an increasing function of P_n and a decreasing function of α_n . That is $\frac{\delta X_n}{\delta P_n} > 0$

⁶Also known as utility function and welfare function.



Fig. 2. Demonstration of the effects of different parameters on the net benefit achieved by an EU by trading its energy with the SFC. As can be seen from the figure, a higher sensitivity α_n leads the EU *n* to achieve its maximum net benefit at a lower p_n . Also, as the surplus of an EU decreases, it achieves its maximum net benefit at a relatively higher price compare to other EUs.

and $\frac{\delta X_n}{\delta \alpha_n} < 0$. Therefore, an EU with higher sensitivity will be prone not to change its selling price p_n significantly.

A graphical representation of the properties of the benefit function X_n is shown in Fig. 2.

It is important to note that for sustainable energy trading in a smart grid system the overall system benefits from trading needs to be profitable. Otherwise, EUs with significantly lower revenue from energy trading would not participate in such trading [15], which will eventually diminish the energy trading market. In this context, the objective of each EU *n* in the system is to interact with the SFC in order to determine a price p_n per unit of energy, within the budget *C* in (1) of the SFC, so as to sell its energy surplus e_n such that the sum $\sum_n X_n$ over all EUs in the system is maximized. Mathematically, the objective can be expressed as

$$\max_{p_n} X(\mathbf{p}) = \max_{p_n} \left[\sum_n \left(P_n p_n - \frac{\alpha_n}{2} p_n^2 - e_n p_n \right) \right], \quad (4)$$

such that $\sum_{n} e_{n}p_{n} \leq C$. Now, to explore how each EU *n* in the proposed system can identify a price p_{n} within the budget *C* in (1) such that their objective in (4) can be attained, we propose a CCG in the next section.

We stress that the proposed problem can also be solved by using other techniques such as dual decomposition or other centralized schemes. In order to solve the problem in a distributed fashion and thus allow the EUs to maintain their privacy (i.e., not to disclose private information like α_n and P_n to the SFC and other EUs in the network), we choose to use a CCG over other centralized techniques. As discussed in [46], sharing such private information not only allows the SFC to control the EUs' energy usage behavior, but also enables the SFC to infer the EUs' private lifestyles, which is a significant privacy concern at present. Hence, certain information, such as α_n (the sensitivity of a user to the change of price), needs to be kept private and should not be shared with the SFC. In addition, although the SFC may solve the proposed problem in a centralized manner if it has access to the private information, the EUs may be concerned that the SFC could modify α_n , and thus distribute the total budget in favor of the EUs that have good relationships with the SFC, which is especially possible if the EUs and the SFC do not trust each other. Therefore, considering the above-mentioned factors, we choose to use a distributed technique such as the proposed CCG that leads to a solution with desirable properties like social optimality and being envy-free, while at the same time preserving the privacy of the EUs.

III. CAKE CUTTING GAME

A. Brief Background

CCGs comprise a branch of game theory that deals with the division of some finite pool of resources in a way that meets certain valuation criteria or objectives of the players splitting the resource [47]. Formally, the cake can be represented as a convex set, which is the total budget $C \subset \mathbb{R}$ of the SFC in this proposed case. Each player *n* will receive an allocation $p_n e_n$ of this *C* by choosing a suitable p_n with the property $p_n e_n \subseteq C$. Now, before proceeding to the design of the proposed game, first we discuss some key properties of a CCG [47], which are relevant to the proposed scheme as follows.

1) The allocation vector $\mathbf{p}^* = [p_1^*, p_2^*, \dots, p_N^*]$, which contains the outcome received by each player $n \in \mathcal{N}$ via distributing the cake *C* is *complete* if

$$\sum_{n} e_n p_n^* = C. \tag{5}$$

2) An allocation is called a *socially optimal* allocation if the allocation has the property

$$X(\mathbf{p}^*) \ge X(\mathbf{p}),\tag{6}$$

where $\mathbf{p} = [p_1^*, p_2^*, \dots, p_n, \dots, p_N^*]$ for any $n \in \mathcal{N}$.

3) An allocation \mathbf{p}^* possesses the property of *Pareto opti*mality if no player with an allocation p_{n1}^* can be better off with a share p_{n1} without hurting at least one other player. Mathematically, an allocation with p_{n1}^* and p_{n2}^* is Pareto optimal, if there exists no other allocation containing p_{n1} and p_{n2} such that

$$X_{n1}(p_{n1}^{*}) \leq X_{n1}(p_{n1}) \wedge X_{n2}(p_{n2}^{*}) \leq X_{n2}(p_{n2}),$$

$$\forall n1 = 1, 2, \dots, N, \exists n2 = 1, 2, \dots, N;$$

$$p_{n1}^{*}, p_{n2}^{*} \in \mathbf{p}^{*}.$$
(7)

 If a *complete allocation* p* of a CCG is socially optimal, it is also Pareto optimal [47].

B. Proposed Game

To decide on the energy trading parameter p_n , each EU in the smart community interacts with the SFC, and we propose a CCG Π to capture this interaction. In the proposed CCG Π , each EU *n* decides on its selling price p_n through the considered game and offers the price to the SFC. The SFC, on the other hand, compares the received price vector **p** from all the EUs and decides whether the total expense is within its budget *C*, i.e., if the expense satisfies (1), and thus decides the solution of the game. Formally, the proposed CCG Π can be whose Jacobean is defined by its strategic form as⁷

$$\Pi = \{\mathcal{N}, \{\mathbf{P}_n\}_{n \in \mathcal{N}}, X\},\tag{8}$$

where \mathcal{N} is the set of players, i.e., EUs, in the game and \mathbf{P}_n is the set of strategies of EU *n*, i.e., $p_n \in \mathbf{P}_n$, $\forall n$, that satisfy (1). In (8), the choice of each EU $p_n \in \mathbf{P}_n$ affects the choices of other EUs in choosing their suitable selling prices due to the presence of (1). Hence, the proposed CCG Π can be considered as a *variational inequality* problem Γ [48], in which the choices of strategies of multiple players are coupled through (1). Hence, variational equilibrium, which is the solution concept of variational inequality problems, can be presumed as the solution of the proposed CCG Π . For the rest of the paper, we will use the term **cake cutting solution (CCS)** to refer to a variational equilibrium of the proposed CCG Π .

Definition 1: Consider the CCG Π in (8), in which X is defined by (4). A set of strategies \mathbf{p}^* constitutes the CCS of the proposed Π if and only if the strategy set satisfies the following inequalities:

$$X(p_n^*, \mathbf{p}_{-n}^*) \ge X(p_n, \mathbf{p}_{-n}^*),$$

$$\forall n \in \mathcal{N}, p_n \in \mathbf{P}_n \ \forall n, \ \sum_n e_n p_n \le C,$$
 (9)

where $\mathbf{p}_{-n} = [p_1, p_n, \dots, p_{n-1}, p_{n+1}, \dots, p_N].$

C. Properties of CCS

In this section, we investigate the properties of the CCS. In particular, we determine whether there exists a socially optimal CCS of the proposed CCG Π . Essentially, a socially optimal solution maximizes the total benefit to all the EUs in the smart grid, and thus is suitable for allocation of *C* in order to maximize the overall system benefit.

Theorem 1: There exists a socially optimal CCS of the proposed CCG Π between the EUs in the smart grid.

Proof: First we note that the proposed CCG Π is a variational inequality problem Γ as we have mentioned earlier. Therefore, the CCG Π can be defined as $\Gamma(\mathbf{P}, \mathbf{Z})$, which can be used to determine a vector $\mathbf{p} \in \mathbf{P} \in \mathbb{R}^n$ such that $\langle \mathbf{Z}(\mathbf{p}^*), \mathbf{p} - \mathbf{p}^* \rangle \ge 0$, $\forall \mathbf{p} \in \mathbf{P}$ [49]. Here,

$$\mathbf{Z} = -\left(\nabla_{p_n} X_n(p_n)\right)_{n \in \mathcal{N}},\tag{10}$$

and **P** is the vector of all strategies of all EUs in the network. The solution of the $\Gamma(\mathbf{P}, \mathbf{Z})$ is the CCS. Now, the pseudogradient of the benefit function (4) is [49]

$$\mathbf{Z} = \begin{bmatrix} \alpha_{1}p_{1} + e_{1} - P_{1} \\ \alpha_{2}p_{2} + e_{2} - P_{2} \\ \vdots \\ \alpha_{N}p_{N} + e_{N} - P_{N} \end{bmatrix},$$
(11)

⁷Since the EUs do not have storage facilities, all EUs with energy surplus will be interested in participating in the game to make revenue as long as the offered price is more than the grid's buying price. Hence, the proposed CCG falls within the example of game theoretic problems where competitive buyers participate in the game to increase their utilities, e.g., studies in [23], [40], and [41].

$$\mathbf{JZ} = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0\\ 0 & \alpha_2 & \cdots & 0\\ \vdots & \vdots & \cdots & \vdots\\ 0 & 0 & \cdots & \alpha_N \end{bmatrix}.$$
 (12)

In (12), α_n for n = 1, 2, ..., N is always positive. Now, by considering the n^{th} leading principal minor \mathbf{JZ}_n of the leading principal sub-matrix⁸ \mathbf{JZ} , it can be shown that \mathbf{JZ}_n is always positive, i.e., $|\mathbf{JZ}_1| > 0$, $|\mathbf{JZ}_2| > 0$ and so on. Therefore, \mathbf{JZ} is positive definite on \mathbf{P} , and thus \mathbf{Z} is strictly monotone. Hence, $\Gamma(\mathbf{P}, \mathbf{Z})$ possesses a unique CCS [48]. Furthermore, due to the presence of the joint constraint (1), the CCS is also the unique global maximizer of (4) [48], which subsequently proves that the CCS is the *socially optimal* solution of the proposed CCG Π .

Whereas the CCG Π is shown to have a socially optimal solution, in order to divide the budget *C* among the EUs in an efficient manner it is also necessary that the budget should be Pareto optimal. We note that in a Pareto efficient allocation no EU can change its strategy without hurting at least one another EU in the network. Therefore, if the allocation is both socially and Pareto optimal, the allocation of price per unit of energy between different EUs will be fair and envy-free. To this end, first we note that the social optimality of the CCS has already been shown in Theorem 1. Therefore, according to Property 4 in Section III-A, demonstrating the existence of a complete allocation of price between the EUs will subsequently establish the Pareto optimality of the CCS.

Now, due to the coupled constraint (1), the Karush-Kuhn-Tucker (KKT) conditions, using the method of Lagrange multipliers [50], for the EU *n*'s choice on p_n in (4) can be defined as [48]

 $P_n - \alpha_n p_n - e_n - \tau_n = 0, \, \tau_n \ge 0,$

and

$$\tau_n\left(\sum_n e_n p_n - C\right) = 0, \ \forall n.$$
⁽¹⁴⁾

Here, τ_n is the Lagrange multiplier for EU *n*. It is important to note that if any strictly monotone variational inequality problem constitutes a CCS such as the proposed case (according to Theorem 1), the multiplier τ_n , $\forall n$ possesses the property $\tau_n > 0$, $\forall n$ [48]. As a consequence, it is clear from Theorem 1 and (14) that for the proposed CCG Π the total allocation of budget between the EUs is equal to *C*, i.e., $\sum_n p_n e_n = C$. Thus, the allocation is *complete*, and consequently the CCS of the proposed CCG Π is Pareto optimal.

Remark 1: Since, the solution of the CCG Π possesses a solution, which is both socially and Pareto optimal, the allocation of price per unit of energy between different EUs will be fair and envy-free.⁹

(13)

⁸The *i*th order principal sub-matrix A_i can be obtained by deleting the last g - i rows and last g - i columns from a $g \times g$ matrix A.

⁹Such an envy-free property allows each EU to trade its surplus energy with the SFC without envying other EUs in the network for the payments they receive, and thus ensures market transparency despite price discrimination and consequently enables such an energy trading market to be sustained.



Algorithm 1 Algorithm to Reach the CCS of the Proposed CCG Π

Another important characteristic of the decision making process of an EU *n* concerning its choice of a price p_n per unit of energy can be explained from (13). Since, $\tau_n > 0 \ \forall n$,

$$p_n < \frac{P_n - e_n}{\alpha_n}.$$
 (15)

Thus, an EU with higher energy surplus and/or higher sensitivity to the choice of price needs to choose a relatively smaller price per unit of energy compared to EUs with lower surplus and/or lower sensitivity in order to reach the socially and Pareto optimal CCS if the proposed CCG Π is adopted.

D. Algorithm

Now, to design an algorithm for the EUs to reach the desired solution of the CCG Π , we first note that the proposed game is a strictly monotone variational inequality problem. Therefore, the CCS can be attained by solving the game through an algorithm that is suitable for solving a monotone variational inequality problem. To this end, we propose to use the S-S method [51], which is shown to be effective to solve monotone variational inequality problems in [15], to solve the proposed CCG Π . Essentially, the S-S method used in this paper is a hyperplane projection method that requires two-way communications between EUs and the SFC in the network to reach the CCS. A geometrical interpretation is used where two projections per iteration are required. For instance, let p^t be the current approximation of the solution of $\Gamma(\mathbf{P}, \mathbf{Z})$. Then, first the projection $r(p^t) = \operatorname{Proj}_{\mathbf{P}}[p^t - \mathbf{Z}(p^t)]$ is computed¹⁰ and a point z^t is searched for in the line segment between p^t and $r(p^t)$ such that the hyperplane $\partial H \triangleq \{p \in \mathbb{R} | \langle \mathbf{Z}(k^t), p - z^t \rangle = 0\}$ strictly separates p^t from the solution \mathbf{p}^* of the problem. Then, once ∂H is constructed, p^{t+1} is computed in iteration t+1

¹⁰Projection $\operatorname{Proj}_{\mathbf{P}}[k] = \arg\min\{||w - k||, w \in \mathbf{P}\}, \forall k \in \mathbb{R}^n [15].$

by projecting onto the intersection of the feasible set **P** with hyperspace $H^t \triangleq \{p \in \mathbb{R} | \langle \mathbb{Z}(k^t), p - z^t \rangle \leq 0\}$, which contains the solution set.

The proposed algorithm is initiated with the announcement of a total budget by the SFC to buy electricity from the connected EUs. The SFC can set the budget using any statistical technique such as a Markov chain model based on historical budget data sets [52]. Upon receiving the information about the budget and determining its own requirement, each EU decides the amount of energy that it wants to sell and submits it to the SFC. Once the budget is set and the surplus of each EU is determined, all the EUs participate in the proposed CCG Π through Algorithm 1 and reach the optimal price vector, i.e., the CCS \mathbf{p}^* , for selling their surplus energy to the SFC. The details of the algorithm in shown in Algorithm 1.

Proposition 1: The algorithm proposed in Algorithm 1 is always guaranteed to reach the CCS of the proposed CCG Π .

Proof: To prove Proposition 1, first we note that the S-S method is based on a hyperplane projection technique [49], which is always guaranteed to converge to a non-empty solution if the variational inequality problem is strictly monotone [48]. It is proven in Theorem 1 that the proposed CCG Π is strictly monotone over the choice of $p_n \forall n$. Therefore, the proposed Algorithm 1 is guaranteed to always reach a non-empty CCS, and thus Proposition 1 is proved.

Note: Please note that the proposed game is a static game and therefore does not consider the effects of changes in parameters across different time slots. Here, we consider a single time instant and keep the entire focus of the study on investigating how the considered energy trading scheme can be conducted in an envy-free environment so as to achieve a socially optimal solution by using the proposed price discrimination technique. Once such price discrimination is established, extending the proposed work to a time-varying



Fig. 3. Demonstration of the convergence of the proposed Algorithm 1 to the CCS of the proposed CCG Π . It is noted from the figure that the price vector reaches its CCS after 8th iterations.

environment is an interesting topic for future work. Note that the electricity price in real-time pricing schemes is decided differently at different times of the day based on the conditions of several parameters such as the demand, electricity generation, and the reserve of energy in the system. Therefore, the proposed scheme has the potential to be incorporated into such a real-time pricing scheme, in which the energy controller may decide to adopt price discrimination at a particular time of the day, whenever it seems beneficial.

IV. CASE STUDY

To show the effectiveness of the proposed scheme, we consider an example in which a number of EUs with energy surplus are participating in supplying energy to the SFC in a time of interest. The energy surplus of each EU to supply to the SFC is assumed to be a uniformly distributed random variable in the range [3.6, 12.25] kWh [15]. The target per unit price P_n for all $n \in \mathcal{N}$ is assumed to be 45 cents, which is in fact equal to the grid's selling price [22]. This value is based on the rationality assumption of the EUs where each EU is willing to charge the SFC as much as the grid for selling its energy. The budget of the SFC is considered to be 1000 cents, which is chosen to maintain the condition $p_n e_n \leq C < P_n e_n$ throughout the simulation process. Further, the choice of C is also chosen such that the price p_n for any EU n does not go below the grid's buying price $p_{g,buy} = 8.00$ cents/kWh [22]. This is necessary to ensure that all EUs are interested in selling to the SFC instead of to the grid. The sensitivity parameter $\alpha_n \forall n$ is chosen randomly from the range [1, 3]. Thus, a consumer with $\alpha_n = 1$ is least sensitive to its choice of price whereas consumers with sensitivity ≈ 3 are considered to be strictly constrained to the price choice. Nevertheless, it is important to note that all parameters used in this study are particular for this example only and may vary according to the needs of the SFC, weather conditions, time of day/year and the energy policy of the particular country.

To that end, in Fig. 3, we first show how the proposed game with 10 EUs converges to its CCS solution when we



Fig. 4. Demonstration of the effect of the total budget of the SFC and the number of EUs that are sharing the budget on the average net benefit achieved by each EU participating in the proposed CCG Π .

adopt Algorithm 1. Firstly, according to this figure, the choice of price of each of the participating EUs reaches its CCS after the 8th iteration of the algorithm. Hence, the speed of the algorithm is reasonable. Secondly, different EUs determine different prices to pay to the SFC for energy trading, which is mainly due to the way that the pricing scheme is designed. Note that although the target price $P_n \forall n$ per unit of energy is considered similar for all EUs, the energy surpluses available to them are different. Also, different EUs have differing sensitivity to the pricing policy. As a consequence, once the CCG Π reaches the CCS, the socially optimal price vector constitutes a different price per unit of energy for each EU as demonstrated in Fig. 3.

It is important to note that the outcome of the proposed CCG Π is significantly affected by the size of the cake of the game, i.e., the total budget available to the SFC, and the number of participating EUs that are sharing the cake among themselves through choosing a suitable price per unit of energy. Now, to show the effect of the number of EUs that are sharing the budget of the SFC through the proposed CCG Π , we consider an example where four different EUs are sharing the SFC's budget in order to maximize their net benefits by choosing a suitable price per unit of energy. The graphical representation of this considered study is shown in Fig. 4. We change the number of EUs from 10 to 40 with an increment of 10, and show the average net benefits achieved by each EU of each respective case in the figure. According to Fig. 4, first we note that as the number of EUs increases, the average benefit achieved by each EU decreases due to sharing the same budget of the SFC. For instance, as the number of EUs increases from 10 to 40, the average benefit for sharing 1000 cents by choosing a suitable price per unit of energy decreases from 1250 to 390, which is approximately a 68% decrement. Essentially, more EUs taking part in energy management enables each EU to take a smaller share of the budget, which subsequently reduces their benefits in energy trading with the same total budget.

Another interesting aspect that can be noticed from the effect of the SFC's budget on the overall energy trading

TABLE II DEMONSTRATION OF THE EFFECT OF THE SFC'S TOTAL BUDGET ON THE PARTICIPATION RATE OF EUS (WITH SURPLUS ENERGY) IN ENERGY TRADING. IT IS CONSIDERED THAT IF ANY EU DOES NOT RECEIVE A MINIMUM PAYMENT OF $p_{g,BUY} = 8.00$ cents/kWh it Does Not TRADE ITS ENERGY WITH THE SFC. THUS, A LOWER BUDGET FOR A LARGE NUMBER OF EUS SUBSEQUENTLY DECREASES THE PERCENTAGE RATE OF THE EUS' PARTICIPATION IN ENERGY TRADING WITH THE SFC. HERE, THE ACTUAL NUMBERS OF EUS THAT PARTICIPATE IN ENERGY TRADING ARE SHOWN WHEREBY THEIR PERCENTAGES COMPARED TO THE TOTAL EUS IN THE NETWORK ARE SHOWN WITHIN THE BRACKETS

Pudgot (conts)	Percentage EU participation			
Buuget (cents)	Total EU =10	Total EU =20	Total EU =30	Total EU =40
1000	10 (100%)	8 (40%)	6 (20%)	0 (0%)
2000	10 (100%)	20 (100%)	18 (60%)	16 (40%)
3000	10 (100%)	20 (100%)	30 (100%)	32 (80%)
4000	10 (100%)	20 (100%)	30 (100%)	40 (100%)

scheme is that the same budget may not be suitable to encourage all the EUs from EU groups of different sizes to trade their energy with the SFC. In this regard, we show the participation rate of EUs from different sizes of EU groups in Table II. It is assumed that if the price per unit of energy that an EU charges the SFC for selling its energy falls below $p_{g,\text{buy}}$, which is considered to be 8.00 cents/kWh for this particular case study, it does not participate in energy trading. This is due to the fact that, as explained in Section II, the expected return from selling energy for the EU becomes very small. Now, according to Table II, for a similar budget C of the SFC, the participation rate of EUs reduces considerably as the number of EUs in a group increases. For instance, for a budget of 1000 cents, 100% EU participation is observed from a group of 10 EUs, whereby the participation rate decreases to 40%, 20% and 0% respectively for EU group size of 20, 30 and 40. Nonetheless, by increasing the budget C, the SFC can encourage more EUs to be involved in the energy trading with the SFC and can reduce its total cost of energy trading. Thus, as shown in Table II, the proposed scheme can essentially assist the SFC in deciding on its budget with a view to increase participation, if appropriate, with the SFC.

Remark 2: It is important to note from the above discussion in Fig. 4 and Table II that a suitable choice of the SFC's budget *C* is critical to successful adaptation of energy management in the community through the proposed CCG Π as this may possibly affect the average net benefit per EU as well as the total cost incurred by the SFC. The proposed scheme has the potential to assist the SFC in deciding on its budget in order to encourage more EUs, if feasible, to take part in energy trading with the SFC.

Furthermore, the choice of price and consequently the obtained net benefit by each EU is also affected by its sensitivity parameter $\alpha_n \forall n$, which we show in Table III. For this case, we assume that all EUs in the system are equally sensitive to their chosen price per unit of energy, i.e., they have the same α_n , $\forall n$, whereby their available surpluses to sell to the SFC are different as in previous examples. We consider five sensitivity parameters including 1.0, 1.5, 2.0, 2.5 and 3.0, where 1.0 refers to the case when the EUs are insensitive to

TABLE III
DEMONSTRATION OF THE EFFECT OF THE SENSITIVITY PARAMETER OF
EUS ON THEIR AVERAGE NET BENEFIT. FOR THIS PARTICULAR
EXAMPLE, ALL 10 EUS ARE CONSIDERED TO POSSESS THE
SAME SENSITIVITY PARAMETER, I.E., ALL EUS HAVE THE
SAME SENSITIVITY TO THEIR CHOSEN PRICE PER UNIT
OF ENERGY. THE AVERAGE NET BENEFIT PER EU
IS COMPARED, I.E., PERCENTAGE DECREMENT
AS SHOWN WITHIN BRACKETS, WITH
THE CASE WHEN ALL EUS
POSSESS $\alpha_n = 1, \forall n$

Sensitivity a _n	Average net benefit (Budget C = 1000 cents)	Average net benefit (Budget <i>C</i> = 2000 cents)
1	521.71	945.00
1.5	486.27 (- 6.6%)	904.6 (- 4.2%)
2.0	466.06 (-10.6%)	881.47 (- 6.7%)
2.5	452.78 (-13.2%)	865.25 (- 8.4%)
3.0	442.93 (-15.1%)	852.48 (-9.8%)

the choice of price per unit of energy and 3 indicates maximum sensitivity of the EU. The average net benefit to the EU at $\alpha_n = 1$ is considered as the baseline and is used to compare how the average net benefit to the EUs varies as their sensitivity is altered in the system. As can be seen in Table III, the average net benefit to the EUs decreases as the sensitivity increases. For a budget of 1000 cents, for instance, as the sensitivity parameter increases from 1 to 3, the average net benefit to the EU reduces by 15.1%. In fact, as the sensitivity increases, the choice of price becomes more restricted for an EU, which consequently reduces its net benefit. A reduction in net benefit with increasing sensitivity is also observed for a budget of 2000 cents. However, for the higher budget, we find two modifications in terms of achieved average net benefit per EU. Firstly, the benefit per EU increases as the budget of the SFC increases, which is explained in Fig. 4. Secondly, the decrement of average net benefit per EU for different sensitivity parameters compared to $\alpha_n = 1$ is relatively lower for a larger budget. For example, as the sensitivity parameter increases from 1 to 3, the average net benefit per EU reduces to 9.8% for a budget of 2000 cents, whereas this reduction is 15.1% for 1000 cents. This is due to the fact that although α_n is considered the same for all EUs, the available surplus of each EU is different, which enables discriminate pricing. Now, based on the available surplus, each EU chooses its price per unit of energy, which increases for a larger budget (as shown in Fig. 4). As a consequence, the difference between net benefits reduces at a higher budget compared to the case of a lower budget.

Having demonstrated some properties of the proposed discriminate pricing scheme, we now show how the proposed scheme can be beneficial to both the EUs and the SFC compared to the traditional case when both parties trade their energy with the grid. To show this comparison, we assume that the selling and buying prices per unit of energy set by the grid are 44 and 8 cents per kWh as discussed earlier in this section. To this end, we first consider in Table IV how much energy the SFC can buy within its budget if it only buys from the grid, compared to the case in which the SFC buys from the users through the proposed scheme. Then, in Table V, we show the total monetary benefit EUs can attain if they choose to sell their surplus to the SFC instead of selling the surplus to the grid.

TABLE IV Demonstration of the Benefit to the SFC in Terms of the Amount of Energy That the SFC can Get From the Proposed Trading Scheme Compared to the Case When the SFC Only Buys Its Energy From the Grid for the Same Budget

	Energy purchased by the SFC		
Trading scheme	Budget 1000 cents	Budget 2000 cents	Budget 3000 cents
Buying from the grid	22.7 kWh	45.5 kWh	68.18 kWh
Proposed scheme (Buying from EUs)	81 kWh	162 kWh	243 kWh
Increase in purchasing energy amount for the proposed case (with same budget)	58.3 kWh	116.54 kWh	174.82 kWh

TABLE V Demonstration of the Benefits to the EUs in Terms of Total Monetary Revenue That the Participating EUs Receive When They Trade Their Surplus Energy With the SFC Compared to Trading Energy With the Grid

Trading scheme	Total revenue to participating EUs			
	10 EUs	20 EUs	30 EUs	
Sell to the grid	648 cents	1296 cents	1944 cents	
Proposed scheme (Sell to the SFC)	1000 cents	2000 cents	3000 cents	
Revenue improvement to the EUs for the proposed scheme	352 cents	704 cents	1056 cents	

In Table IV, we show the amount of energy that an SFC can buy from EUs if the proposed scheme is adopted as the SFC changes its budget from 1000 cents to 2000 and then 3000 cents. We note that as the SFC increases its budget, more EUs would be interested in taking part in energy trading with the SFC (e.g., as shown in Table II), which subsequently increases the amount of energy that the SFC can obtain from the participating EUs. A similar increment in purchasing energy is also observed in the case when the SFC buys its energy from the main grid. However, for each budget, the energy that the SFC can buy from the EUs is considerably larger than the amount that the SFC can buy from the grid. For instance, for a budget of 1000 cents, the SFC can buy 58.3 kWh more energy through the proposed scheme compared to buying exclusively from the grid. This is due to the fact that the grid price is generally very high [36]. Hence, for a fixed budget C, the SFC can buy relatively smaller amounts of energy from the grid. As a consequence, the SFC manages to buy considerably more energy with the same budget. This phenomenon is observed for all considered SFC's budgets as shown in Table IV.

In Table V, we show how the participating EUs in the proposed scheme can benefit in terms of total revenue that they can receive from trading their surplus energy to the SFC compared to selling to the grid. To this end, we first note that the proposed energy scheme through CCG Π is *complete* as discussed in Section III-B. Therefore, the total revenue that the participating EUs receive, for 10, 20 and 30 EUs in the network, becomes equal to the considered budgets of 1000, 2000, and 3000 cents respectively. By contrast, as the EUs trade their energy with the grid the total revenue reduces significantly because of the lower buying price per unit of energy from the grid. For example, with 10 EUs participating in the CCG Π , the total revenue that they receive is equal to the

1000 cents budget of the SFC. Nonetheless, as the EUs trade their energy with the main grid, due to a lower per unit price of 8 cents/kWh, the total revenue that the participating EUs receive reduces to 648 cents, which is 352 cents less than the proposed scheme for the considered system parameters. The performance improvement for the proposed case is better for higher numbers of EUs in the network.

Remark 3: It is clear from Table IV and Table V that the proposed scheme is beneficial for both the SFC and the participating EUs in the smart grid network for the system parameters considered in the given case studies. Hence, the proposed scheme has the potential to be adopted in practical systems in order to benefit all participating entities.

V. CONCLUSION

This paper has demonstrated a viable method to discriminate price per unit of energy between different energy users in a smart grid system when the EUs sell their surplus energy to a shared facility controller. A cake cutting game has been proposed to leverage the generation of discriminate pricing within a constrained budget of the SFC. To study the fairness of the proposed scheme, it has been shown that the CCG can be modeled as a variational inequality problem that possesses the solution of the game, i.e., the cake cutting solution. The properties of the CCS have been studied and the existence of a socially optimal solution, which is also Pareto optimal, has been validated. An algorithm has been proposed that can be adopted by each EU interacting with the SFC in a distributed manner and the convergence of the algorithm to the optimal CCS has been confirmed. Finally, the properties of the game have been studied, and the advantages of discriminate pricing for both the SFC and the EUs have been demonstrated via simple comparisons with energy trading with the main grid.

An important extension of the proposed scheme would be to establish a relationship between the budget of the SFC and the total number of EUs participating in the energy trading with a view to enabling efficient price discrimination, e.g., by using an interactive Stackelberg game with imperfect information of the total budget. Another potential extension is to conduct studies that determine when such a discriminate pricing technique can be used as a real-time pricing scheme. In addition, extending the proposed scheme to a time-varying environment is another interesting topic for future work.

REFERENCES

- X. Fang, S. Misra, G. Xue, and D. Yang, "Smart grid—The new and improved power grid: A survey," *IEEE Commun. Surveys Tuts.*, vol. 14, no. 4, pp. 944–980, Oct. 2012.
- [2] Y. Cao, T. Jiang, and Q. Zhang, "Reducing electricity cost of smart appliances via energy buffering framework in smart grid," *IEEE Trans. Parallel Distrib. Syst.*, vol. 23, no. 9, pp. 1572–1582, Sep. 2012.
- [3] L. Yu, T. Jiang, and Y. Cao, "Energy cost minimization for distributed Internet data centers in smart microgrids considering power outages," *IEEE Trans. Parallel Distrib. Syst.*, vol. 26, no. 1, pp. 120–130, Jan. 2015.
- [4] W. Tushar, C. Yuen, B. Chai, D. B. Smith, and H. V. Poor, "Feasibility of using discriminate pricing schemes for energy trading in smart grid," in *Proc. IEEE Glob. Commun. Conf. (GLOBECOM)*, Austin, TX, USA, Dec. 2014, pp. 1–7.

- [6] H. Asano, S. Sagai, E. Imamura, K. Ito, and R. Yokoyama, "Impacts of time-of-use rates on the optimal sizing and operation of cogeneration systems," *IEEE Trans. Power Syst.*, vol. 7, no. 4, pp. 1444–1450, Nov. 1992.
- [7] S. de la Torre, J. M. Arroyo, A. J. Conejo, and J. Contreras, "Price maker self-scheduling in a pool-based electricity market: A mixed-integer LP approach," *IEEE Trans. Power Syst.*, vol. 17, no. 4, pp. 1037–1042, Nov. 2002.
- [8] T. Jiang, Y. Cao, L. Yu, and Z. Wang, "Load shaping strategy based on energy storage and dynamic pricing in smart grid," *IEEE Trans. Smart Grid*, vol. 5, no. 6, pp. 2868–2876, Nov. 2014.
- [9] K. Ma, G. Hu, and C. J. Spanos, "Distributed energy consumption control via real-time pricing feedback in smart grid," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 5, pp. 1907–1914, Sep. 2014.
- [10] J. H. Yoon, R. Baldick, and A. Novoselac, "Dynamic demand response controller based on real-time retail price for residential buildings," *IEEE Trans. Smart Grid*, vol. 5, no. 1, pp. 121–129, Jan. 2014.
- [11] C. Wan, Z. Xu, Y. Wang, Z. Y. Dong, and K. P. Wong, "A hybrid approach for probabilistic forecasting of electricity price," *IEEE Trans. Smart Grid*, vol. 5, no. 1, pp. 463–470, Jan. 2014.
- [12] P. Wang, H. Zareipour, and W. D. Rosehart, "Descriptive models for reserve and regulation prices in competitive electricity markets," *IEEE Trans. Smart Grid*, vol. 5, no. 1, pp. 471–479, Jan. 2014.
- [13] M. Rahimiyan, L. Baringo, and A. J. Conejo, "Energy management of a cluster of interconnected price-responsive demands," *IEEE Trans. Power Syst.*, vol. 29, no. 2, pp. 645–655, Mar. 2014.
- [14] Z.-Y. Xu, W.-S. Xu, W.-H. Shao, and Z.-Y. Zeng, "Real-time pricing control on generation-side: Optimal demand-tracking model and information fusion estimation solver," *IEEE Trans. Power Syst.*, vol. 29, no. 4, pp. 1522–1535, Jul. 2014.
- [15] W. Tushar, J. A. Zhang, D. B. Smith, H. V. Poor, and S. Thiébaux, "Prioritizing consumers in smart grid: A game theoretic approach," *IEEE Trans. Smart Grid*, vol. 5, no. 3, pp. 1429–1438, May 2014.
- [16] Y. Liu *et al.*, "Peak-to-average ratio constrained demand-side management with consumer's preference in residential smart grid," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 6, pp. 1084–1097, Dec. 2014.
- [17] N. U. Hassan, M. A. Pasha, C. Yuen, S. Huang, and X. Wang, "Impact of scheduling flexibility on demand profile flatness and user inconvenience in residential smart grid system," *Energies*, vol. 6, no. 12, pp. 6608–6635, Dec. 2013.
- [18] Y. Liu *et al.*, "Electricity cost minimization for a microgrid with distributed energy resource under different information availability," *IEEE Trans. Ind. Electron.*, vol. 62, no. 4, pp. 2571–2583, Apr. 2015.
- [19] N. U. Hassan, Y. I. Khalid, C. Yuen, and W. Tushar, "Customer engagement plans for peak load reduction in residential smart grids," *IEEE Trans. Smart Grid*, vol. 6, no. 6, pp. 3029–3041, Nov. 2015.
- [20] N. U. Hassan *et al.*, "Framework for minimum user participation rate determination to achieve specific demand response management objectives in residential smart grids," *Elsevier Int. J. Elect. Power Energy Syst.*, vol. 74, pp. 91–103, Jan. 2016.
- [21] W. Tushar, C. Yuen, S. Huang, D. B. Smith, and H. V. Poor, "Cost minimization of charging stations with photovoltaics: An approach with EV classification," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 1, pp. 156–169, Aug. 2015.
- [22] W. Tushar *et al.*, "Three-party energy management with distributed energy resources in smart grid," *IEEE Trans. Ind. Electron.*, vol. 62, no. 4, pp. 2487–2498, Apr. 2015.
- [23] W. Tushar, W. Saad, H. V. Poor, and D. B. Smith, "Economics of electric vehicle charging: A game theoretic approach," *IEEE Trans. Smart Grid*, vol. 3, no. 4, pp. 1767–1778, Dec. 2012.
- [24] J. Bartash. (Aug. 7, 2015). This is the Only Major U.S. Industry Cutting Jobs. [Online]. Available: http://www.marketwatch.com/story/ this-is-the-only-major-us-industry-cutting-jobs-2015-08-07?mod=MW_ story_recommended_default&Link=obnetwork
- [25] E. Yudovina and G. Michailidis, "Socially optimal charging strategies for electric vehicles," *IEEE Trans. Autom. Control*, vol. 60, no. 3, pp. 837–842, Mar. 2015.
- [26] J. Robinson, Economics of Imperfect Competitions. New York, NY, USA: Macmillan, 1933.
- [27] R. Schmalensee, "Output and welfare implications of monopolistic third-degree price discrimination," *Amer. Econ. Rev.*, vol. 71, no. 1, pp. 242–247, Mar. 1981.
- [28] H. R. Varian, "Price discrimination and social welfare," Amer. Econ. Rev., vol. 75, no. 4, pp. 870–875, Sep. 1985.

- [29] A. I. Czerny and A. Zhang, "Airport congestion pricing when airlines price discriminate," *Elsevier Transp. Res. B Methodol.*, vol. 65, pp. 77–89, Jul. 2014.
- [30] M. Grennan, Price Discrimination and Bargaining: Empirical Evidence From Medical Devices, Wharton School, Univ. Pennsylvania, Philadelphia, PA, USA, Feb. 2012, pp. 1–53. [Online]. Available: SSRM:http://ssrn.com/abstract=1721145
- [31] I. Hendel and A. Nevo, "Intertemporal price discrimination in storable goods markets," *Amer. Econ. Rev.*, vol. 103, no. 7, pp. 2722–2751, Dec. 2013.
- [32] S. Cowan, "Third-degree price discrimination and consumer surplus," J. Ind. Econ., vol. 60, no. 2, pp. 333–345, Jun. 2012.
- [33] K. Tsakalozos et al., "Flexible use of cloud resources through profit maximization and price discrimination," in Proc. IEEE Int. Conf. Data Eng. (ICDE), Hanover, Germany, Apr. 2011, pp. 75–86.
- [34] Wholesale Solar. (2012). Grid-Tied Solar Power Systems With Battery. [Online]. Available: http://www.solarchoice.net.au/ blog/which-electricity-retailer-is-giving-the-best-solar-feed-in-tariff/
- [35] SolarRay. (2015). Grid-Tie Package Systems Without Batteries. [Online]. Available: http://www.solarray.com/CompletePackages/ Grid-Tie-No-Batteries_T.php
- [36] E. McKenna and M. Thomson, "Photovoltaic metering configurations, feed-in tariffs and the variable effective electricity prices that result," *IET Renew. Power Gener.*, vol. 7, no. 3, pp. 235–245, May 2013.
- [37] D. Barclay et al., "Regulated retail electricity prices 2014-15," Queensland Competition Authority, Brisbane, QLD, Australia, May 2014. [Online]. Available: http://www.qca.org.au/getattachment/ 25696fbc-b4ed-42c4-8d16-fd3efddb563e/Final-Determination.aspx
- [38] SolarMatters. (2015). Feed-in Tariff for Grid Connected Solar Power System. [Online]. Available: http://www.energymatters.com.au/ rebates-incentives/feedintariff/#queensland
- [39] Z. Fan, "Distributed demand response and user adaptation in smart grids," in *Proc. IFIP/IEEE Int. Symp. Integr. Netw. Manage.*, Dublin, Ireland, May 2011, pp. 726–729.
- [40] P. Samadi, A.-H. Mohsenian-Rad, R. Schober, V. W. S. Wong, and J. Jatskevich, "Optimal real-time pricing algorithm based on utility maximization for smart grid," in *Proc. IEEE Int. Conf. Smart Grid Commun.* (*SmartGridComm*), Gaithersburg, MD, USA, Oct. 2010, pp. 415–420.
- [41] B. Chai, J. Chen, Z. Yang, and Y. Zhang, "Demand response management with multiple utility companies: A two-level game approach," *IEEE Trans. Smart Grid*, vol. 5, no. 2, pp. 722–731, Mar. 2014.
- [42] Z. Han, D. Niyato, W. Saad, T. Başar, and A. Hjøorungnes, *Game Theory in Wireless and Communication Networks*. Cambridge, U.K.: Cambridge Univ. Press, 2014.
- [43] T. Başar and R. Srikant, "Revenue-maximizing pricing and capacity expansion in a many-users regime," in *Proc. IEEE Int. Conf. Comput. Commun.*, vol. 1, New York, NY, USA, Jun. 2002, pp. 294–301.
- [44] Y. Wang, W. Saad, Z. Han, H. V. Poor, and T. Başar, "A game-theoretic approach to energy trading in the smart grid," *IEEE Trans. Smart Grid*, vol. 5, no. 3, pp. 1439–1450, May 2014.
- [45] M. Fahrioğlu and F. L. Alvarado, "Designing cost effective demand management contracts using game theory," Dept. Elect. Comput. Eng., Univ. Wisconsin, Madison, WI, USA, Tech. Rep. PSerc 98–30, Dec. 1998. [Online]. Available: http://pserc.wisc.edu/documents/publications/ papers/1998_general_publications/reportmurat.pdf
- [46] F. G. Mármol, C. Sorge, O. Ugus, and G. M. Pérez, "Do not snoop my habits: Preserving privacy in the smart grid," *IEEE Commun. Mag.*, vol. 50, no. 5, pp. 166–172, May 2012.
- [47] P. Bartal and I. Nagy, "Game theoretic approach for achieving optimum overall efficiency in DC/DC converters," *IEEE Trans. Ind. Electron.*, vol. 61, no. 7, pp. 3202–3209, Jul. 2014.
- [48] F. Facchinei and C. Kanzow, "Generalized Nash equilibrium problems," 40R, vol. 5, no. 3, pp. 173–210, Sep. 2007.
- [49] D. Arganda, B. Panicucci, and M. Passacantando, "A game theoretic formulation of the service provisioning problem in cloud systems," in *Proc. Int. World Wide Web Conf.*, Hyderabad, India, Apr. 2011, pp. 177–186.
- [50] D. P. Bertsekas, Nonlinear Programming. Belmont, MA, USA: Athena Scientific, 1995.
- [51] M. V. Solodov and B. F. Svaiter, "A new projection method for variational inequality problems," *SIAM J. Control Optim.*, vol. 37, no. 3, pp. 767–776, 1999.
- [52] A. Shamshad, M. A. Bawadi, W. M. A. W. Hussin, T. A. Majid, and S. A. M. Sanusi, "First and second order Markov chain models for synthetic generation of wind speed time series," *Energy*, vol. 30, no. 5, pp. 693–708, Apr. 2005.



Wayes Tushar (S'06–M'13) received the B.Sc. degree in electrical and electronic engineering from the Bangladesh University of Engineering and Technology, Bangladesh, in 2007, and the Ph.D. degree in engineering from Australian National University, Australia, in 2013. Currently, he is a Research Scientist with the Singapore University of Technology and Design (SUTD)-MIT International Design Center, SUTD, Singapore. Prior joining SUTD, he was a Visiting Researcher with National ICT Australia, ACT, Australia. He was also a

Visiting Student Research Collaborator with the School of Engineering and Applied Science, Princeton University, NJ, USA, in Summer 2011. His research interest includes signal processing for distributed networks, game theory, and energy management for smart grids. He was a recipient of the two best paper awards, both as the first author, at the Australian Communications Theory Workshop 2012 and the IEEE International Conference on Communications 2013.



David B. Smith (S'01–M'04) received the B.E. degree in electrical engineering from the University of New South Wales, Australia, in 1997, and the M.E. (research) and Ph.D. degrees in telecommunications engineering from the University of Technology Sydney, in 2001 and 2004, respectively. He is a Senior Researcher with National ICT Australia (NICTA), and an Adjunct Fellow with Australian National University (ANU). He has been with NICTA and ANU since 2004. He has also had a variety of industry experience in electrical and

telecommunications engineering. He has published over 90 technical refereed papers and made various contributions to IEEE standardization activity. His research interests are in wireless body area networks, game theory for distributed networks, mesh networks, disaster tolerant networks, radio propagation, MIMO wireless systems, space-time coding, antenna design, and also in distributed optimization for smart grid. He was a recipient of four conference best paper awards.



H. Vincent Poor (S'72–M'77–SM'82–F'87) received the Ph.D. degree in EECS from Princeton University in 1977. From 1977 to 1990, he was with the faculty of the University of Illinois at Urbana–Champaign. Since 1990, he has been on the faculty of Princeton University, where he is the Michael Henry Strater University Professor of Electrical Engineering and the Dean of the School of Engineering and Applied Science. His research interests are in the areas of stochastic analysis, statistical signal processing, and information theory

Chau Yi from Na in 2000 a Postde Bell Lab Assistant Universi a Senio for Info

Chau Yuen received the B.Eng. and Ph.D. degrees from Nanyang Technological University, Singapore, in 2000 and 2004, respectively. In 2005, he was a Postdoctoral Fellow with Lucent Technologies Bell Laboratories, Murray Hill. He was a Visiting Assistant Professor with Hong Kong Polytechnic University in 2008. From 2006 to 2010, he was a Senior Research Engineer with the Institute for Infocomm Research, Singapore. He joined the Singapore University of Technology and Design as an Assistant Professor from 2010. He was a recipient

of the IEEE Asia-Pacific Outstanding Young Researcher Award in 2012. He serves as an Associate Editor for the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY.

and their applications in wireless networks and related fields such as social networks and smart grid. Among his publications in these areas is the recent book *Mechanisms and Games for Dynamic Spectrum Allocation* (Cambridge University Press, 2014).

Dr. Poor received a Guggenheim Fellowship in 2002, and the IEEE Education Medal in 2005. Recent recognition of his work includes the 2014 URSI Booker Gold Medal, the 2015 EURASIP Athanasios Papoulis Award, the 2016 John Fritz Medal, and honorary doctorates from several universities in Europe and Asia. In 1990, he served as the President of the IEEE Information Theory Society, and from 2004 to 2007, he served as the Editor-in-Chief of the IEEE TRANSACTIONS ON INFORMATION THEORY. He is a member of the National Academy of Engineering and the National Academy of Sciences, and a foreign member of Academia Europaea and the Royal Society. He is also a fellow of the American Academy of Arts and Sciences, the Royal Academy of Engineering, U.K., and the Royal Society of Edinburgh.