# Three-User Cooperative NOMA Transmission 

Zhaoxi Fang ${ }^{\oplus}$, Member, IEEE, Jiang Hu, Yingzhi Lu, and Wei Ni ${ }^{\ominus}$, Senior Member, IEEE


#### Abstract

This letter presents a new downlink cooperative nonorthogonal multiple access (NOMA) transmission scheme to serve three users within only two time slots. The scheme involves a base station (BS), two direct-link users and one indirect-link user. The BS sends superposed signals to the two direct-link users which decode and forward the signals to the indirect-link user in an alternating fashion. Closed-form expression is derived for the sum-rate of the proposed scheme and a simple expression of sum-rate is also derived in the high signal-to-noise ratio (SNR) region. In the presence of strong inter-user interference (IUI), the proposed scheme can be applied by having each direct-link user to detect the IUI first and then cancel the IUI to decode the signals destined for the indirect-link user and itself. Simulation results show that the scheme with configurable decoding orders at the users is able to achieve a higher sum-rate than existing orthogonal multiple access (OMA) based alternatives.


Index Terms-Cooperative NOMA, decode-and-forward, asymptotic analysis.

## I. Introduction

NON-ORTHOGONAL multiple access (NOMA) is a promising multiple access technique for the fifthgeneration (5G) mobile communication systems, and has increasingly received attentions [1]-[5]. In a NOMA system, multiple users (devices) can be served at the same time and/or frequency resource by appropriately assigning the transmit powers for the users, and the spectral efficiency can be substantially improved, as compared with conventional orthogonal multiple access (OMA) schemes [6], [7].

Recently, the NOMA technique has been extended to cooperative and relay-assisted communication systems [8]-[10]. In [5], the authors proposed a cooperative NOMA scheme with a dedicated half-duplex decode-and-forward (DF) relay, where a base station (BS) broadcasts superposition coded signals to a direct-link user and a dedicated relay in the first phase, and the half-duplex relay forwards re-encoded signals to both the direct-link user and an indirect-link user, which is further away from the BS, in the second phase. A similar cooperative NOMA scheme with an amplify-and-forward (AF)

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Z. Fang is with the School of Mechanical and Electrical Engineering, Shaoxing University, Shaoxing 312000, China, and also with the College of Information and Intelligent Engineering, Zhejiang Wanli University, Ningbo 315100, China (e-mail: zhaoxifang@gmail.com).
$\mathrm{J} . \mathrm{Hu}$ and $\mathrm{Y} . \mathrm{Lu}$ are with the College of Information and Intelligent Engineering, Zhejiang Wanli University, Ningbo 315100, China (e-mail: hujiang@zwu.edu.cn; luyingzhi@zwu.edu.cn).
W. Ni is with the Commonwealth Scientific and Industrial Research Organization, Sydney, NSW 2122, Australia (e-mail: wei.ni@csiro.au).

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Fig. 1. The proposed cooperative NOMA transmission scheme for a threeuser downlink system, where the left-hand side (LHS) and the right-hand side (RHS) depict the operations of the BS and users at the odd and even time slots, respectively.
relay was considered in [8], and a closed-form expression for the exact outage probability was derived.

For cooperative NOMA with multiple half-duplex relays, a two-stage relay selection algorithm was proposed in [9] and the corresponding achievable diversity order was analyzed. Instead of dedicated relays, the users with better channel conditions were selected as relays to help the other users [10]. However, these existing NOMA transmission schemes suffer from rate losses, as the half-duplex relays cannot receive and transmit signals at the same time. Additional time slots are required for the relays to forward signals to the users with poor channel conditions.
Motivated to improve the spectral efficiency of half-duplex relay assisted NOMA systems, we propose a new cooperative NOMA transmission scheme to accomplish transmitting to three users in TWO time slots. (In contrast, existing cooperative NOMA systems [5] can only serve two users in two time slots due to the half-duplex constraint.) In the proposed scheme, a BS transmits packets to two direct-link users and, through which, to an indirect-link user beyond the coverage of the BS. The two direct-link users receive superposition coded signals from the BS, while acting as the relays to forward re-encoded signals to the indirect user in an alternating manner. With such cooperative NOMA transmission, each of the three users is able to receive its own data within the two time slots, as shown in Fig. 1. A closed-form expression is derived for the sum-rate of the proposed scheme with a much simpler asymptotic expression of sum-rate provided in high signal-tonoise ratio (SNR) regions. In the presence of strong inter-user interference (IUI), the proposed can also be applied by having each direct-link user to detect the IUI first and then cancel the IUI to decode the signals destined for the indirect-link user and itself one after the other. Numerical results confirm that the proposed cooperative NOMA scheme with configurable decoding orders at the users can outperform the OMA based alternatives markedly in terms of sum-rate.

## II. Proposed Three-User Cooperative NOMA

Fig. 1 depicts the proposed downlink cooperative NOMA scheme, where there are a BS and three users $T_{k}, k=1,2,3$,
all operating in a half-duplex mode. We assume users $T_{1}$ and $T_{2}$ are close to the BS and can receive signals form the BS directly; while the third user $T_{3}$ is far from the BS with no direct link, as considered as in [5], [8]. The channel from the BS to user $T_{k}$ is $h_{k}(k=1,2)$. The channel between users $T_{i}$ and $T_{j}$ is $g_{i j}, i, j \in\{1,2,3\}$. All the channels are independent Rayleigh fading channels with $\mathcal{E}\left(\left|h_{k}\right|^{2}\right)=G_{k}, k=1,2$, and $\mathcal{E}\left(\left|g_{i j}\right|^{2}\right)=G_{i j}, i, j \in\{1,2,3\}$. In this letter, we consider the case that the BS only has the knowledge of the large-scale fading of every radio link involved; or in other words, the average channel gain or the channel quality indicator (CQI), as considered in many practical systems.

In the proposed scheme, during the odd-numbered time slots $2 t+1, t=0,1,2, \ldots$, the BS transmits superposition coded signals to user $T_{1}$, while user $T_{2}$ forwards signals to the indirect-link user $T_{3}$. In the even-numbered time slots $2 t, t=1,2, \ldots$, the BS transmits superposition coded signals to user $T_{2}$, and user $T_{1}$ forwards the signals received in the previous time slot to user $T_{3}$.

1) Odd-Numbered Time Slots: At the odd-numbered time slot $2 t+1$, the superposition coded transmit signal of the BS is given by
$x_{B}(2 t+1)=\sqrt{\alpha_{1} P_{B}} s_{1}(2 t+1)+\sqrt{\alpha_{2} P_{B}} s_{3}(2 t+1)$,
where $s_{1}(2 t+1)$ and $s_{3}(2 t+1)$ denote the unit-power signal destined for users $T_{1}$ and $T_{3}$, respectively, $\alpha_{1}$ and $\alpha_{2}$ are the power allocation factors for the two users with $\alpha_{1}+\alpha_{2}=1$ and $\alpha_{1}<\alpha_{2}$, and $P_{B}$ is the transmission power of the BS.

At the odd-numbered time slot $2 t+1$, user $T_{2}$ also acts as a relay and forwards the decoded signal to user $T_{3}$. As will be shown later in (7), user $T_{2}$ has received mixed signals from the BS and $T_{1}$ at the previous time slot $2 t$. Suppose that the signals from the BS are successfully decoded. The transmit signal of user $T_{2}$ is given by

$$
\begin{equation*}
x_{2}(2 t+1)=\sqrt{P_{2}} \hat{s}_{3}(2 t) \tag{2}
\end{equation*}
$$

where $P_{2}$ is the transmit power of user $T_{2}$, and $\hat{s}_{3}(2 t)$ denotes the signals destined for $T_{3}$ and correctly detected by user $T_{2}$. Therefore, the received signals at users $T_{1}$ and $T_{3}$ can be respectively expressed as

$$
\begin{align*}
& y_{1}(2 t+1)=h_{1} x_{B}(2 t+1)+g_{21} x_{2}(2 t+1)+n_{1}(2 t+1) ;  \tag{3}\\
& y_{3}(2 t+1)=g_{23} x_{2}(2 t+1)+n_{3}(2 t+1) \tag{4}
\end{align*}
$$

where $n_{k}(2 t+1), k=1,3$, denotes the additive white Gaussian noise (AWGN) at user $T_{k}$ with variance $\sigma^{2}$.

By substituting (1) and (2) into (3), the received signal at user $T_{1}$ at the odd-numbered time slots can be expressed as

$$
\begin{align*}
y_{1}(2 t+1)= & \sqrt{\alpha_{1} P_{B}} h_{1} s_{1}(2 t+1)+\sqrt{\alpha_{2} P_{B}} h_{1} \\
& \times s_{3}(2 t+1)+\sqrt{P_{2}} g_{21} \hat{s}_{3}(2 t)+n_{1}(2 t+1), \tag{5}
\end{align*}
$$

where the first and second terms on the right-hand side (RHS) of (5) are the desired signals. The third term is the IUI which can degrade the performance. Based on the received signal in (5), successive interference cancellation (SIC) can be used to detect the desired signals. Assuming that the two direct-link users have better channel conditions than the indirect-link user, then the decoding order is $s_{3}(2 t+1) \rightarrow s_{1}(2 t+1)$ at user $T_{1}$. Specifically, user $T_{1}$ first decodes $s_{3}(2 t+1)$, and then
subtracts it from the received signal to detect its own signal $s_{1}(2 t+1)$ by treating $\hat{s}_{3}(2 t)$ as interference. ${ }^{1}$
2) Even-Numbered Time Slots: The signal transmission and detection processes at the even-numbered time slots are similar to those at the odd-numbered time slots. At the even-numbered time slot $2 t$, the transmit signal of the BS is given by

$$
\begin{equation*}
x_{B}(2 t)=\sqrt{\alpha_{1} P_{B}} s_{2}(2 t)+\sqrt{\alpha_{2} P_{B}} s_{3}(2 t), \tag{6}
\end{equation*}
$$

where $s_{3}(2 t)$ is the signal destined for user $T_{3}$, detected by user $T_{2}$ at the current time slot, and will be forwarded to user $T_{3}$ at the next time slot.

The transmit signal of user $T_{1}$ is $x_{1}(2 t)=\sqrt{P_{1}} \hat{s}_{3}(2 t-$ $1)$, where $\hat{s}_{3}(2 t-1)$ denotes the detected signal at user $T_{1}$ based on the received signal at the previous odd-numbered time slot $2 t-1$. The received signals at users $T_{2}$ and $T_{3}$ are respectively given by

$$
\begin{align*}
y_{2}(2 t)= & \sqrt{\alpha_{1} P_{B}} h_{2} s_{2}(2 t)+\sqrt{\alpha_{2} P_{B}} h_{2} s_{3}(2 t) \\
& +\sqrt{P_{1}} g_{12} \hat{s}_{3}(2 t-1)+n_{2}(2 t)  \tag{7}\\
y_{3}(2 t)= & g_{13} x_{1}(2 t)+n_{3}(2 t), \tag{8}
\end{align*}
$$

where $n_{k}(2 t), k=2,3$, is the AWGN with variance $\sigma^{2}$.
The decoding and re-encoding operations of user $T_{2}$ at the even-numbered time slots are similar to those of user $T_{1}$ at the odd-numbered time slots. Based on (7), user $T_{2}$ first decodes $s_{3}(2 t)$, and then subtracts the decoded signal from the received signal to detect the desired signal $s_{2}(2 t) . \hat{s}_{3}(2 t-1)$ is treated as interference. The detected and re-encoded signal $\hat{s}_{3}(2 t)$ will be forwarded to user $T_{3}$ at the next odd-numbered time slot, as shown in (2).

## III. Performance Analysis

## A. Achievable Rates

Define $\rho_{B}=P_{B} / \sigma^{2}$ and $\rho_{k}=P_{k} / \sigma^{2}, k=1,2$. From (5) and (7), we see that the direct-link user $T_{1}$ (or $T_{2}$ ) receives superposition coded signals from the BS and interference from user $T_{2}$ (or $T_{1}$ ). At user $T_{k}, k=1,2$, we assume the signal destined for user $T_{3}$ is decoded and canceled successfully, by exploiting the merit of NOMA techniques [1]. Then, the achievable rate for the direct-link user $T_{k}$ is given by

$$
\begin{equation*}
R_{k}=\frac{1}{2} \log _{2}\left(1+\frac{\alpha_{1}\left|h_{k}\right|^{2} \rho_{B}}{1+\left|g_{\bar{k} k}\right|^{2} \rho_{\bar{k}}}\right) \tag{9}
\end{equation*}
$$

where $\bar{k}=3-k$, and $k=1,2$.
Note that $T_{3}$ receives the signals at both the odd- and even-numbered time slots. Consider the transmission from the BS to user $T_{3}$ via user $T_{1}$. From (5), the received signal-to-interference-plus-noise ratio (SINR) at user $T_{1}$ for the detection of $s_{3}(2 t+1)$ is given by

$$
\begin{equation*}
\gamma_{3, B \rightarrow T_{1}}=\frac{\alpha_{2}\left|h_{1}\right|^{2} \rho_{B}}{\alpha_{1}\left|h_{1}\right|^{2} \rho_{B}+\left|g_{21}\right|^{2} \rho_{2}+1} \tag{10}
\end{equation*}
$$

For the relayed transmission from user $T_{1}$ to user $T_{3}$, the received SINR at user $T_{3}$ is given by $\gamma_{3, T_{1} \rightarrow T_{3}}=\left|g_{13}\right|^{2} \rho_{1}$. As user $T_{1}$ operates in the DF mode for forwarding the signal

[^0]$s_{3}(2 t+1)$, the achievable rate from the BS to user $T_{3}$ via user $T_{1}$ is given by
\[

$$
\begin{equation*}
R_{3, \mathrm{odd}}=\frac{1}{2} \log _{2}\left(1+\min \left(\gamma_{3, B \rightarrow T_{1}}, \gamma_{3, T_{1} \rightarrow T_{3}}\right)\right) \tag{11}
\end{equation*}
$$

\]

Likewise, the achievable data rate for the relayed transmission from the BS to user $T_{3}$ via user $T_{2}$ is given by

$$
\begin{equation*}
R_{3, \text { even }}=\frac{1}{2} \log _{2}\left(1+\min \left(\gamma_{3, B \rightarrow T_{2}}, \gamma_{3, T_{2} \rightarrow T_{3}}\right)\right) \tag{12}
\end{equation*}
$$

where $\gamma_{3, T_{2} \rightarrow T_{3}}=\left|g_{23}\right|^{2} \rho_{2}$ is the received SINR at user $T_{3}$, and $\gamma_{3, B \rightarrow T_{2}}$ denotes the received SINR at user $T_{2}$ for decoding the signal $s_{3}(2 t)$, as given by

$$
\begin{equation*}
\gamma_{3, B \rightarrow T_{2}}=\frac{\alpha_{2}\left|h_{2}\right|^{2} \rho_{B}}{\alpha_{1}\left|h_{2}\right|^{2} \rho_{B}+\left|g_{12}\right|^{2} \rho_{1}+1} . \tag{13}
\end{equation*}
$$

The sum-rate for the proposed three-user cooperative NOMA system can be given by

$$
\begin{equation*}
R_{\mathrm{sum}}=R_{1}+R_{2}+R_{3, \text { odd }}+R_{3, \text { even }} \tag{14}
\end{equation*}
$$

## B. Sum-Rate Performance

Theorem 1: The closed-form expression for the sum-rate of the proposed cooperative NOMA scheme is given by

$$
\begin{align*}
& \mathcal{E}\left(R_{\text {sum }}\right)=\sum_{k=1}^{2} \frac{\log _{2}(e)}{2\left(1-c_{k}\right)}\left[F_{1}\left(\frac{1}{\alpha_{1} G_{k} \rho_{B}}\right)-F_{1}\left(\frac{1}{G_{\bar{k} k} \rho_{\bar{k}}}\right)\right] \\
& +\sum_{k=1}^{2} \frac{\log _{2}(e)}{2} e^{\frac{1}{\rho_{k} G_{k 3}}}\left[E_{1}\left(\frac{1}{\rho_{k} G_{k 3}}\right)-E_{1}\left(\frac{1}{\alpha_{1} \rho_{k} G_{k 3}}\right)\right],( \tag{15}
\end{align*}
$$

where $\bar{k}=3-k, c_{1}=G_{21} \rho_{2} / \alpha_{1} G_{1} \rho_{B}, c_{2}=$ $G_{12} \rho_{1} / \alpha_{1} G_{2} \rho_{B}, E_{1}(x)=\int_{x}^{+\infty}\left(e^{-t} / t\right) d t$ is the exponential integral function, and $F_{1}(x)=e^{x} E_{1}(x)$.

Proof: First, consider the transmission rate of user $T_{1}$. The CDF of the received SINR of user $T_{1}, \gamma_{1}=\frac{\alpha_{1}\left|h_{1}\right|^{2} \rho_{B}}{1+\left|g_{21}\right|^{2} \rho_{2}}$, is given by

$$
\begin{align*}
F_{\gamma_{1}}(x) & =P\left(\left|h_{1}\right|^{2} \leq \frac{x\left(1+\left|g_{21}\right|^{2} \rho_{2}\right)}{\alpha_{1} \rho_{B}}\right) \\
& =1-\frac{1}{1+c_{1} x} e^{-\frac{x}{\alpha_{1} G_{1} \rho_{B}}} \tag{16}
\end{align*}
$$

Next, the achievable rate of user $T_{1}$ is given by

$$
\begin{align*}
\mathcal{E} & \left(R_{1}\right)=\int_{0}^{+\infty} \frac{1}{2} \log _{2}(1+x) f_{\gamma_{1}}(x) d x \\
& =\frac{\log _{2}(e)}{2\left(1-c_{1}\right)} \int_{0}^{+\infty}\left(\frac{1}{1+x}-\frac{c_{1}}{1+c_{1} x}\right) e^{-\frac{x}{\alpha_{1} G_{1} \rho_{B}}} d x \\
& =\frac{\log _{2}(e)}{2\left(1-c_{1}\right)}\left[F_{1}\left(\frac{1}{\alpha_{1} G_{1} \rho_{B}}\right)-F_{1}\left(\frac{1}{G_{21} \rho_{2}}\right)\right], \tag{17}
\end{align*}
$$

where $f_{\gamma_{1}}(x)$ is the PDF of $\gamma_{1}$, and the last step is due to the fact that $\int_{0}^{+\infty} e^{-a x} /(b+x) d x=e^{a b} E_{1}(a b)$ ([11, eq. (5.1.28)]).

Likewise, the rate of user $T_{2}$ can be obtained in closed form, as given by

$$
\begin{equation*}
\mathcal{E}\left(R_{2}\right)=\frac{\log _{2}(e)}{2\left(1-c_{2}\right)}\left[F_{1}\left(\frac{1}{\alpha_{1} G_{2} \rho_{B}}\right)-F_{1}\left(\frac{1}{G_{12} \rho_{1}}\right)\right] . \tag{18}
\end{equation*}
$$

We proceed to analyze the rate of user $T_{3}$. Given that the IUI is much weaker than the received signal power at users
$T_{1}$ and $T_{2}$ in practice, $\gamma_{3, B \rightarrow T_{1}}$ can be approximated by $\gamma_{3, B \rightarrow T_{1}} \simeq \frac{\alpha_{2}}{\alpha_{1}}$. As a result, $R_{3, \text { odd }}$ in (11) in the high SNR region can be approximated by

$$
\begin{equation*}
R_{3, \text { odd }} \simeq \frac{1}{2} \log _{2}\left(1+\min \left(\frac{\alpha_{2}}{\alpha_{1}}, \gamma_{T_{1} \rightarrow T_{3}}\right)\right) \tag{19}
\end{equation*}
$$

The corresponding rate of user $T_{3}$ in the odd-numbered time slots can be evaluated as

$$
\begin{align*}
\mathcal{E} & \left(R_{3, \text { odd }}\right)=\mathcal{E}\left[\frac{1}{2} \log _{2}\left(1+\min \left(\frac{\alpha_{2}}{\alpha_{1}}, \gamma_{T_{1} \rightarrow T_{3}}\right)\right)\right] \\
& =\int_{0}^{\frac{\alpha_{2}}{\alpha_{1}}} \frac{1}{2} \log _{2}(1+x) f_{\gamma_{T_{1} \rightarrow T_{3}}}(x) d x \\
& +\int_{\frac{\alpha_{2}}{\alpha_{1}}}^{+\infty} \frac{1}{2} \log _{2}\left(1+\frac{\alpha_{2}}{\alpha_{1}}\right) f_{\gamma_{T_{1} \rightarrow T_{3}}}(x) d x \\
& =\frac{\log _{2}(e)}{2} e^{\frac{1}{\rho_{1} G_{13}}}\left[E_{1}\left(\frac{1}{\rho_{1} G_{13}}\right)-E_{1}\left(\frac{1}{\alpha_{1} \rho_{1} G_{13}}\right)\right], \tag{20}
\end{align*}
$$

where $f_{\gamma_{T_{1} \rightarrow T_{3}}}(x)=\frac{1}{\rho_{1} G_{13}} e^{-x / \rho_{1} G_{13}}$ is the PDF of $\gamma_{T_{1} \rightarrow T_{3}}$.

Likewise, for the data rate of user $T_{3}$ in the even-numbered time slots, we have

$$
\begin{align*}
\mathcal{E}\left(R_{3, \text { even }}\right)= & \frac{\log _{2}(e)}{2} e^{\frac{1}{\rho_{2} G_{23}}} \\
& \times\left[E_{1}\left(\frac{1}{\rho_{2} G_{23}}\right)-E_{1}\left(\frac{1}{\alpha_{1} \rho_{2} G_{23}}\right)\right] . \tag{21}
\end{align*}
$$

By adding up (17), (18), (20) and (12), we obtain (15).

## C. Asymptotic Performance Under High SNR

In the following, we analyze the asymptotic performance in the high SNR region, i.e., $\rho_{B}, \rho_{1}$ and $\rho_{2} \rightarrow+\infty$. From [11, eq. (5.1.11)], when $x \rightarrow 0, E_{1}(x)$ can be approximated by $E_{1}(x) \simeq-\mathcal{C}_{0}-\ln (x)$, where $\mathcal{C}_{0}=0.5772$ is the Euler constant. Hence, $\mathcal{E}\left(R_{1}\right)$ in (17) can be approximated by

$$
\begin{equation*}
\mathcal{E}\left(R_{1}\right) \simeq \frac{\log _{2}(e)}{2\left(1-c_{1}\right)} \ln \left(\frac{1}{c_{1}}\right) \tag{22}
\end{equation*}
$$

The high-SNR approximations of $\mathcal{E}\left(R_{2}\right), \mathcal{E}\left(R_{3, \text { odd }}\right)$ and $\mathcal{E}\left(R_{3, \text { even }}\right)$ can be derived in a similar way: $\mathcal{E}\left(R_{2}\right) \simeq$ $\frac{\log _{2}(e)}{2\left(1-c_{2}\right)} \ln \left(\frac{1}{c_{2}}\right)$, and $\mathcal{E}\left(R_{3, \text { odd }}\right) \simeq \mathcal{E}\left(R_{3, \text { even }}\right) \simeq \frac{1}{2} \log _{2}\left(\frac{1}{\alpha_{1}}\right)$.
Then, the sum-rate in the high SNR region can be approximated by

$$
\begin{equation*}
\bar{R}_{\mathrm{sum}} \simeq \sum_{k=1}^{2} \frac{1}{2\left(1-c_{k}\right)} \log _{2}\left(\frac{1}{c_{k}}\right)+\log _{2}\left(\frac{1}{\alpha_{1}}\right) \tag{23}
\end{equation*}
$$

Remark 1: Recall that $c_{k}=G_{\bar{k} k} \rho_{\bar{k}} / \alpha_{1} G_{k} \rho_{B}$. From (23), it can be seen that the IUI has a dominating impact on the sum-rate in the high SNR region. When the transmit power of the BS is much higher than the users, i.e., $\rho_{B} \gg \rho_{k}, k_{-}=1,2$, the sum-rate in (23) can be further approximated by $\bar{R}_{\text {sum }} \simeq$ $\frac{1}{2} \log _{2}\left(\frac{G_{1} G_{2} \rho_{B}^{2}}{G_{21} G_{12} \rho_{1} \rho_{2}}\right)$, which suggests that the sum-rate in the high SNR region is independent of the power allocation factors $\alpha_{i}, i=1,2$, in the proposed scheme.

Remark 2: In the presence of strong IUI, we can change the decoding orders at the two direct-link users to improve


Fig. 2. Sum-rate versus BS's Tx power for the proposed scheme.
the sum-rate of the proposed approach. Specifically, when the IUI between the direct-link users $T_{1}$ and $T_{2}$ is weak (i.e., the average inter-user channel gain is lower than a threshold), the decoding orders at the users are as described in Section II. When the IUI is strong (i.e., the average inter-user channel gain is higher than the threshold), each direct-link user decodes the IUI from the other direct-link user first, and then cancels the IUI to decode the signals destined for the indirect-link user $T_{3}$ and itself one after the other. Given the new decoding orders and the average channel gains of the links involved, the sum-rate of the proposed scheme can be analyzed in the same way as in Section III-B.

## IV. Simulation Result

We consider a symmetric system that the channel gains from the BS to the direct-link users are set to be $G_{1}=G_{2}=$ -30 dB , while the channel gains between the users are set to be $G_{13}=G_{23}=-40 \mathrm{~dB}$ and $G_{12}=G_{21}=-50 \mathrm{~dB}$. The noise power is -60 dBm , and different transmit powers of the BS and the users are tested. The power allocation factors at the BS are $\alpha_{1}=0.1$ and $\alpha_{2}=0.9$.

Fig. 2 shows the sum-rate of the proposed NOMA scheme under various BS power budgets. The transmit power of the two direct-link users are set to $P_{1}=P_{2}=0 \mathrm{dBm}$. The decoding orders at the two direct-link users remain unchanged in this figure, as described in Section III-A. The proposed schemes are compared with a NOMA based benchmark scheme extended from [5] (labeled as "NOMA benchmark") and a conventional TDMA based DF relaying scheme. For the NOMA benchmark scheme, at the first time slot, the BS sends superposition coded signals to user $T_{1}$, which forwards signal to user $T_{3}$ at the second slot. At the third slot, the BS communicates with $T_{2}$ directly. The transmit power and the power allocation factors of the BS in the NOMA benchmark scheme are set up in the same way as they are in the proposed cooperative NOMA transmission scheme. The transmit power of the user $T_{1}$ doubles that in the cooperative NOMA scheme, so that the total energy consumptions of the NOMA-based mechanism and the proposed cooperative NOMA scheme are consistent. In the TDMA relaying scheme, the two direct-link users $T_{1}$ and $T_{2}$ receive signals from the BS and forward their decoded signals to the indirect-link user $T_{3}$ at different time slots. Fig. 2 shows that the analytical result (15) can accurately quantify the proposed cooperative NOMA scheme. The proposed scheme outperforms the other two schemes, and


Fig. 3. Sum-rate versus the inter-user channel gain.
the gaps become increasingly large with the growth of the BS power.

Fig. 3 shows the sum-rates of the proposed cooperative NOMA schemes, where the inter-user channel gain between the direct-link users $T_{1}$ and $T_{2}$ grows from -80 dB to -20 dB . The transmit powers of the BS and the users are set to be $P_{B}=30 \mathrm{dBm}$ and $P_{1}=P_{2}=0 \mathrm{dBm}$, respectively. The sum-rates of the cooperative NOMA scheme with fixed decoding order (as described in Section III-A.) and with configurable decoding order (as described in Remark 2) are both plotted in the figure. The threshold for the change of the decoding order is -50 dBm in the latter, which is specified in the way to be described shortly. We can see that the performance of the proposed cooperative NOMA scheme degrades as the IUI grows. The cooperative NOMA transmission scheme with fixed decoding order can be worse than other two benchmark schemes if the IUI between users $T_{1}$ and $T_{2}$ is strong (i.e., $G_{12} \geq-35 \mathrm{~dB}$ ). Among all these schemes, the cooperative NOMA scheme with configurable decoding orders achieves the best performance and outperforms the benchmarks markedly, by changing the decoding order to cancel the IUI when the received IUI is strong. It is noteworthy that we keep $\alpha_{1}$ and $\alpha_{2}$ unchanged when changing the decoding orders, in order to decouple the gain of changing the orders (or canceling the strong IUI) from the gain of power allocation. The sum-rate of the proposed algorithm can be further improved by optimizing the value of $\alpha_{1}$ (and $\alpha_{2}=1-\alpha_{1}$ ) given the average channel gains of the links, e.g., based on one-dimensional search.

In Fig. 3, we also plot the sum-rate of the proposed cooperative NOMA scheme, when each direct-link user always starts by decoding the IUI, followed by the signals destined for the indirect-link user $T_{3}$ and itself. This is referred to as "Coop. NOMA with IUI first". The intersection of this curve with the curve of the cooperative NOMA scheme with fixed decoding orders helps specify the threshold of the IUI, i.e., -50 dBm , at which the proposed cooperative NOMA scheme changes the decoding orders.

In Fig. 4, we investigate the impact of the users' transmit powers on the sum-rates of the proposed cooperative NOMA transmission scheme. The total transmit power of the BS and the users is $P_{T}=2 P_{B}+P_{1}+P_{2}=40 \mathrm{dBm}$. The transmit powers of the two users are set equal: $P_{1}=P_{2}=P$. In the figure, we see that the achievable sum-rate of the cooperative NOMA scheme with fixed decoding orders decreases with the transmit power of the users when $P>-10 \mathrm{dBm}$. This is


Fig. 4. Sum-rate versus users' Tx power for the proposed scheme.


Fig. 5. Sum-rate versus the power allocation factor $\alpha_{1}$.
due to the fact that the achievable rate of the indirect-link user $T_{3}$ is subjected to the channels from the BS to the two direct-link users $T_{1}$ and $T_{2}$, especially when $P$ is large. The increasing transmit powers of the direct-link users can degrade the data rates of the users, due to the increasingly strong IUI. Nevertheless, the proposed cooperative NOMA transmission scheme is still able to achieve a higher sum-rate by changing the decoding order at the direct-link users.

Finally, we simulate the proposed cooperative NOMA scheme under different power allocation factors, as shown in Fig. 5. The transmit powers of the BS and the users are set to be $P_{B}=30 \mathrm{dBm}$ and $P_{1}=P_{2}=0 \mathrm{dBm}$, respectively. The other simulation parameters remain the same as those in Fig. 2. It is shown in Fig. 5 that the achievable sum-rates of all the tested NOMA techniques remain almost unchanged
across a wide range of the power allocation factors in high SNR regions, which confirms our analysis.

## V. Conclusion

This letter proposed a new cooperative NOMA transmission scheme for a three-user downlink system. Closed-form expressions were derived for the sum-rate of the proposed scheme and the asymptotic behavior was studied in high SNR regions. By configuring the decoding order of the direct-link users, the proposed cooperative NOMA scheme can operate in the presence of strong IUI and outperform existing techniques markedly in terms of sum-rate.

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[^0]:    ${ }^{1}$ It is possible that the IUI is strong when the two direct-link users are close. In such case, the proposed scheme can be applied by changing the decoding order at the two direct-link users, as will be discussed in Section III-C.

