Abstract—This article considers the problem of passive tracking of a noncooperating target indoors. We develop a novel system to localize a moving target using asynchronous self-localizing sniffer nodes, which passively listen to WiFi signals transmitted by the target. The proposed system uses only the time-difference-of-arrival between multipath components (multipath TDoA) at each receiver. This does not require phase synchronization. We develop two novel localization algorithms; one uses batch processing and the other is online. We also design a novel multipath association algorithm. A custom-designed hardware platform is developed to prototype the proposed system. The accuracy of the proposed system is verified experimentally. For signal-to-noise ratio of 10 dB, both proposed algorithms achieve target localization accuracies better than 40 cm with probability 0.95 without needing any knowledge of target or sniffer locations. If the location of one sniffer node is known, the accuracy improves to 15 cm.

Index Terms—Channel state information (CSI), IEEE 802.11ac signals, indoor localization and tracking, multipath TDoA, particle filtering, passive listening.

I. INTRODUCTION

DROP-AND-PLAY wireless sniffing devices can play a critical role in quickly acquiring situational awareness from inaccessible environments. For example, mapping interiors of buildings from the outside, where video and radar are blocked by walls. In defense, security, and antiterrorism applications the sniffers are required to be passive, which restricts their ability to accurately synchronize. In addition, they may not be able to be accurately located either. In such situations, the state-of-the-art triangulation based on wireless signals can be highly inaccurate due to significant multipath propagations and limitations on sniffer placement, which results in a large geometric dilution of precision (GDOP) [1].

This article develops a new system of drop-and-play passive wireless sniffers which is able to track an indoor noncooperating mobile target that is emitting wireless signals, while also performing self localization of the sniffers, without requiring synchronization between the target and sniffers, or between sniffers. The algorithm assumes knowledge of reflector (i.e., wall) geometries. Specifically, each sniffer measures the channel state information (CSI) from signals transmitted by the target, and estimates the time difference of arrival (TDoA) between multipath components (multipath TDoA), thus allowing practical nonsynchronized node deployments not possible with previous triangulation approaches. By combining the knowledge of room geometry and multipath TDoA estimates, the location of the transmitter is inferred, along with the locations of the sniffers themselves.

Custom-designed sniffer hardware of the proposed system is developed in the unlicensed 5.8 GHz WiFi band. None of the existing commercial-off-the-shelf (COTS) IEEE 802.11ac chipsets grants access to CSI necessary to estimate multipath TDoA. A software-defined radio (SDR) platform has been developed based on AD9361 chips, stitching two relatively narrow-band channel frequency responses (CFRs) of AD9361 chips with bandwidths of 50 MHz to a wideband of 80 MHz accounting for a whole IEEE 802.11ac channel, and measuring the channels of IEEE 802.11ac signals in real-time.

The key contributions of this article are as follows.
1) We propose two algorithms to passively locate a transmitter in a multipath environment, given knowledge of room geometry. One algorithm employs a batch processing approach to jointly estimate the target path and the sniffers locations. The other algorithm employs particle filtering to track the target in an online mode, improving the estimation accuracy over time.
2) We propose an algorithm to associate the measured multipath TDoA components with the reflectors/walls.
3) We experimentally validate the performance of the proposed algorithms in real-time, using a custom designed hardware platform.

In the absence of a priori knowledge of any target or sniffer locations, both batch and online processing algorithms achieve target localization accuracies better than 40 cm with probability greater than 0.95 for signal-to-noise ratio (SNR) 10 dB in an environment layout with 3 walls. With knowledge of the location of only one sniffer node, both the proposed algorithms achieve localization accuracies better than 15 cm with probability greater than 0.95 under the same SNR. The accuracy of the online processing algorithm is slightly better than the batch processing algorithm since the online processing algorithm uses partial...
knowledge of target motion. The results of the experiments conducted in an environment layout with one wall show that using the online processing algorithm, a localization accuracy of 30 cm can be achieved for all target locations. Also, submeter level localization accuracies are achieved for all the sniffer locations by both algorithms in the experiments. Overall, the online processing algorithm shows superior error performances compared to the batch processing algorithm.

The remainder of this article is organized as follows. Section II provides a summary of the background and related work. In Section III, the system description is provided and the passive localization and tracking problem is formulated. Our proposed multipath association algorithms are presented in Section IV and in Section V, we present the proposed localization algorithms. The numerical results and the experimental results are provided in Sections VI and VII, respectively, and Section VIII concludes the article.

The notation used in the article are as follows. Boldface uppercase letters represent matrices and boldface lower case letters represent vectors, $(\cdot)^T$ and $(\cdot)^{-1}$ represent the transpose and the inverse of a matrix, respectively, $\|\cdot\|$ is the Euclidean norm of a vector, $A!$ is the factorial of $A$. $E(\cdot)$ is the expectation, $[A]_{i,j}$ is the $(i,j)$th element of matrix $A$ and $[a,b]$ is the set of integers from $a$ to $b$ including $a$ and $b$ which are integers with $a < b$.

II. BACKGROUND AND RELATED WORK

The ubiquitous usage of WiFi systems indoors makes it a candidate technology for implementing indoor localization. Wireless indoor localization systems typically use either received signal strength (RSS) measurements or CSI [2]. There are many RSS-based schemes, including WiFiSLAM [3] and WiFi GraphSLAM [4], and they have the benefit of a simple measurement process, however they all suffer in multipath environments. The accuracy of RSS-based localization schemes would also be heavily penalized by the multiplicatively coupled errors caused by shadowing [5]. CSI-based schemes inherently capture information about multipath, and so perform better in most indoor scenarios. In this article we take a CSI-based approach.

Another characterisation of localization systems developed using WiFi signals is into two broad categories, triangulation-based techniques, and fingerprinting-based techniques. In triangulation-based techniques [6]–[9], the estimates of the time-of-arrival (ToA) and/or the angle-of-arrival (AoA) of multipath signal components obtained from frequency-domain CSI are used to identify the line-of-sight (LoS) signal component between the target and the anchor nodes and to suppress the multipath signals. The target is localized with the aid of 3 or more anchor nodes. The Chronos system [6] uses the estimates of ToA, the ArrayTrack system [7] uses the estimates of AoA and the SpotFi system [8] uses the estimates of both ToA and AoA to implement triangulation. Xiong and Jamieson [7] and [8] used multiple signal classification (MUSIC) algorithm [10] to estimate propagation parameters of multipath components. The estimation of ToA requires the synchronization between the target and the receivers. Therefore, ToA estimates are not suitable for target tracking through passive listening. To estimate AoA accurately, large antenna arrays are required [7].

Generally, the fingerprinting-based approaches (e.g., [11]–[13]) operate in two phases. In the training phase, a fingerprinting database of the indoor area is generated by applying machine learning techniques on frequency-domain CSI collected at multiple locations. This fingerprinting database is used to predict the location of a target in the same indoor area in the localization phase. For each new indoor environment, the fingerprinting database needs to be re-trained, which is not desirable in rapid ad hoc deployments.

The accuracy of indoor localization can be further enhanced if multipath components are associated with the room geometry instead of suppressing them [14]. In this approach, multipath signal components are treated as being transmitted from virtual transmitters (VT), which correspond to the reflective walls/scatterers located in the indoor environment. Since the signals transmitted by the VTs are used, this approach enables a reduction in the number of receiver nodes required for localization. In [15], the lower bounds for the problem of localizing a receiver node using multipath geometry were derived when the room layout is known.

Using the estimates of ToA of each multipath component, the multipath-geometry-based localization of the receiver nodes and the walls was formulated as a convex optimization problem in [16]. The joint problem of tracking of a moving receiver and the localization of the walls using multipath signals was studied in [17] and [18], where the joint tracking and localization problem was formulated as a Bayesian estimation problem. Estimates of both ToA and AoA of each multipath component were used in both systems.

The estimates of TDoA between multipath components (multipath TDoA) enable the implementation of localisation and tracking in practical scenarios where it is not possible to have synchronization/cooperation between wireless nodes. Multipath TDoA, which does not depend on synchronization, is not to be confused with conventional TDoA where two or more synchronous receiver nodes are used for localization [19].

In [20], joint localization and mapping was implemented for ultra-wideband (UWB) systems using the estimates of multipath TDoA and AoA, where the motion of the target was assumed to be known. The system in [20] required a large antenna array at the receiver to estimate the AoA. Furthermore, the computation time and the memory requirement of the localization algorithm in [20] increased exponentially with the number of sniffer nodes used. In contrast to [20], we use single-antenna sniffers to obtain CSI since our system uses only multipath TDoA for tracking and localization.

III. SYSTEM DESCRIPTION AND THE PASSIVE LOCALIZATION PROBLEM

Consider the application scenario given in Fig. 1. There are $L$ passive sniffer nodes ($L = 6$ in Fig. 1) located outside an office building and there is a target IEEE 802.11 transmitter moving inside the building. The sniffer nodes passively listen to the signals transmitted by the target. The sniffer locations are given
The walls in the environment are denoted by $w$. Each sniffer node receives the direct signal and the signals using multipath signals. Fig. 1. Application scenario for localization of a target and the sniffer locations using multipath signals.

by $r_j$, $j \in [1, L]$ and the indoor environment consists of $N_W$ walls. Each wall acts as a partial transmitter and a partial reflector. Each sniffer node receives the direct signal and the signals reflected by walls. It is assumed that the multipath components other than the first-order reflections and scattered signals are insignificant. The walls in the environment are denoted by $W_k$, $k \in [1, N_W]$.

We express all the locations using two-dimensional coordinates. A discrete-time measurement system is considered where multipath TDoA measurements of the target are obtained in $T$ time instances. At measurement time instance $t$, for a given target location $I(t)$, $t \in [1, T]$, the reflected signal can be considered as being transmitted from a VT which is the mirror image of $I(t)$ against Wall 1, as shown in Fig. 1. Similarly, we can identify the respective virtual transmitter associated with Walls 2 and 3.

We exploit frequency-domain CSI obtained by the sniffer nodes for tracking. With $N$ equally-spaced subcarriers in the orthogonal frequency-division multiplexing (OFDM) signal, the estimated CSI in the subcarrier $n$ at sniffer $r_j$ for the packet $m$ received from the target at $I(t)$ is

$$H(n,t)_{m,j} = \sum_{l=0}^{P_t} h_{m,j}(t) \exp(-2\pi \mathcal{J}(f_0 + n\Delta f)) \times (\tau^j_l(t) + \beta_{m,j}(t)) + w_{m,j}(n,t)$$  \hspace{1cm} (1)

where $P_t$ is the number of significant multipath components in the signal received by the sniffer $j$ at time $t$, $\tau^j_l(t)$ and $h_{m,j}(t)$ are the propagation delay of the $l$th multipath component and the associated complex channel gain, respectively, $\beta_{m,j}(t)$ is the packet detection delay at the sniffer $j$, $\mathcal{J} = \sqrt{-1}$, $f_0$ is the starting frequency of the OFDM signal, $\Delta f$ is the subcarrier spacing of the OFDM signal and $w_{m,j}(n,t)$ is the complex Gaussian noise in subcarrier $n$ at sniffer $j$ at time $t$. Note that $\beta_{m,j}(t)$ is the same for all the multipath signal components of a given packet at a given sniffer node, and varies across packets.

For practical reasons, it is assumed that the target is not synchronized with the sniffer nodes and there is no synchronization between the sniffer nodes. Hence, time-of-arrival (ToA) or conventional TDoA between different sniffer nodes [19] is not available. Instead, we use multipath TDoA for localization, which does not require any type of clock synchronization as explained below.

A. Multipath TDoA

For the scenario given in Fig. 1, each wall can be represented by a hyperplane, with any point $p$ that lies in the wall $W_k$ satisfying

$$w_k^T p + b_k = 0, \quad k \in [1, N_W]$$  \hspace{1cm} (2)

where $w_k \in \mathbb{R}^2$ and $b_k$ is a scalar. In this article, it is assumed that the locations of the walls are known, i.e., $w_k$ and $b_k$ are known for all $k$. The location of the VT which corresponds to the target location $I(t)$ and wall $W_k$ is

$$\alpha^k(t) = I(t) - \frac{2(w_k^T I(t) + b_k)}{\parallel w_k \parallel^2} W_k.$$  \hspace{1cm} (3)

Assuming the signal reflected from wall $W_k$ is the $l$th multipath signal component received at sniffer $j$, the corresponding multipath TDoA due to the the target located at $I(t)$ is

$$\Delta \tau^j_l(t) = \frac{1}{C} \left( \parallel \alpha^k(t) - r_j \parallel - \parallel I(t) - r_j \parallel \right), \quad l \in [1, P_j(t)]$$  \hspace{1cm} (4)

where $C$ is the speed of propagation of the electromagnetic waves. From (4), it is evident that to estimate multipath TDoA, we do not require any synchronization/cooperation between the transmitter and the sniffer, since the delay is measured with respect to the first multipath signal component. Furthermore, the estimation of multipath TDoA is not vulnerable to the variations of packet detection delays since the packet detection delay is cancelled out due to the subtraction in (4).

B. Super-Resolution Estimation of Multipath TDoA

MUSIC algorithm [10] is used to estimate the delays $\{\tau^j_l(t)\}_{l \in [1,P_j(t)]}$ in (1) from the frequency domain CSI. Note that $l = 0$ corresponds to the direct signal component. Multipath TDoA values can be obtained by $\Delta \tau^j_l(t) = \tau^j_l(t) - \tau^0_l(t)$, $l \in [1, P_j(t)]$. It should be noted that the estimates of $\tau^j_l(t)$ contain the packet detection delay as well.

The forward–backward correlation matrix [21] of each packet received is used as an estimate of the autocorrelation matrix, i.e., a covariance matrix is estimated for each packet received by each receiver separately. This ensures that the estimate of delays is not vulnerable to packet detection delays.

C. Multipath Association

Multipath association expresses the mapping between each significant multipath component and the index of the wall which generated the given multipath component. For the multipath TDoA measurement of multipath component $l$ received at sniffer $j$ in time $t$, the association rule can be expressed as $A^j_l(t) = k$, where $k$ is the index of the wall which generates the $l$th multipath component. For $T$ measurement time instances and $L$ sniffers, there are $TL$ total transmitter–receiver pairs, and accurate estimation of multipath association for each multipath component at each transmitter–receiver pair is crucial for accurate localization results. In this article, reliable estimations of multipath associations of each target-sniffer link is obtained using the algorithm proposed in Section IV.
D. The Impact of Symmetry in the Environment on Multipath Association

A prominent challenge in the multipath association is the symmetry in the wall layout, which would lead to estimation ambiguities. In the example shown in Fig. 2, the target and sniffer locations of Orientation 2 can be obtained by rotating Orientation 1 by 180° around the axis of symmetry and vice versa. Orientation 1 generates identical multipath TDoA values to the Orientation 2 in the absence of noise. However, the multipath associations are different for the two orientations. The sniffer nodes do not receive signals reflected by Wall 4. Hence, there is only one axis of symmetry for the wall layout given in Fig. 2. The number of equivalent (equally possible) associations for a given set of multipath TDoA observations is equal to the number of axes of symmetry in the reflective wall layout. With $S$ axes of symmetry in the wall layout, the multipath association for the $l$th multipath signal component received for the link between target at $I(t)$ and sniffer at $r_j$ is denoted by $A_j^{l,s}(t)$, where $s \in [1,S]$.

E. Measurement Model

Let $\Delta_J^2(t)$ be an estimate of $\Delta_J^2(t)$ given in (4). The noise in CSI estimates given in (1) is Gaussian distributed. Note that MUSIC is a nonlinear operation. Therefore, we do not have an exact model for the noise in multipath TDoA estimates $\Delta_J^2(t)$. In this article, the noise in multipath TDoA estimates is modeled as Gaussian distributed with mean zero and variance $\sigma^2$, which is assumed to be the same for all measurements.

To estimate $\sigma$ for a given SNR, we randomly place a target and sniffers in multiple known locations and collect a large number of packets at each sniffer for each target location. Multipath TDoA estimates are obtained for each packet separately using the MUSIC algorithm. $\sigma$ is the estimated standard deviation of multipath TDoA estimates, averaged over all the target and sniffer locations, and the number of multipath components.

IV. FINDING multipath ASSOCIATIONS

The multipath association problem is to find the wall index $A_j^{l,s}(t)$ for $l \in [1,P_j(t)], s \in [1,S], j \in [1,L]$, and $t \in [1,T]$. Exhaustive search can be used to find the multipath associations. In exhaustive search, the optimization problem in (6) is solved for all possible multipath associations and the objective function value at convergence are found for each possible association.
The multipath associations that result in $S$ smallest objective function values are selected as the equivalent set of multipath associations.

Consider an environment with $N_W$ reflective walls. For multipath TDoA-based localization, we are only interested in measurements with at least one multipath component. However, in multipath TDoA-based localization, we are only interested in measurements with at least one multipath component. Therefore, the total number of searches required to obtain the multipath association of all target-sniffer pairs using pure exhaustive search is $\prod_{j=1}^{L} \prod_{t=1}^{T} \lambda_j(t)$. Form (7), $N_W \leq \lambda_j(t) \leq N_W^!$. Therefore, the total number of searches in exhaustive search is between $(N_W!)^{LT}$ and $(N_W^!)^{LT}$ and it is not feasible to complete the exhaustive search within a practically useful time for large $LT$. We provide a computationally feasible, approximate solution for this problem in Section IV-A.

### A. Proposed Multipath Association Algorithms

We seek to tackle the issues of computational complexity and ambiguities due to the symmetry in the wall layout in the multipath association algorithms proposed here. We will consider two cases. In the first case, we will not use knowledge of any target or sniffer locations. Therefore, we will need to account for the ambiguities arising from symmetry, as discussed in Section III-D. In the second case, we will assume that the location of one sniffer is known. This sniffer node can be used as an anchor node which prevents the rotation of the system around the axis of symmetry.

The multipath association algorithm proposed for the first case is presented in Algorithm 1. Here, the algorithm segments the $L$ sniffer locations into $\frac{L}{2}$ nonoverlapping groups, each with size $D_L$. Similarly, the $T$ target locations are partitioned into $\frac{T}{2}$ nonoverlapping groups, each with size $D_T$. The sniffer group indices and the target group indices are given by the sets $G_L = [1, \frac{L}{2}]$ and $G_T = [1, \frac{T}{2}]$, respectively. This algorithm requires prior knowledge of the number of axes of symmetry $S$ in the wall layout, which is easily obtained since the wall locations are known.

In this algorithm, first Target Group 1 ($g_T = 1$) from $G_T$ and Sniffer Group 1 ($g_L = 1$) from $G_L$ are selected. Within this small group of target-sniffer pairs, exhaustive search is used to find the associations by solving the optimization problem in (8) shown at the bottom of this page for all possible associations, which is up to $(N_W^!)^{LT}$. Multipath associations which results in $S$ smallest objective function values at convergence are selected. Moreover, the target and sniffer locations obtained for the $S$ selected multipath associations are recorded. These recorded target and sniffer locations are used as prior knowledge in subsequent steps where the optimization is carried out for the cases where $1 < g_T \leq \frac{T}{2}$ and $1 < g_L \leq \frac{L}{2}$. This algorithm outputs multipath associations for all target-sniffer pairs for all $S$ symmetry orientations.

Now we consider the second case where there is an anchor sniffer node whose location is assumed to be known. Without loss of generality, the location of Sniffer 1 $(r_1)$ is known. The locations of other sniffers and the target locations are unknown. In this scenario, the multipath association solution can be obtained using Algorithm 2. In this algorithm, the sniffer groups...
Algorithm 1: Proposed Multipath Association Algorithm - When All the Target and Sniffer Locations are Unknown.

Input:
- Multipath TDoA estimates for $L$ sniffers obtained at $T$ measurement time instances.
- The number of axes of symmetry in the wall layout $S$.

Algorithm Steps:
1) Let $D_T$ a positive integer with $D_T \geq 2$ and $\frac{T}{D_T}$ is an integer. Segment the $T$ measurement time instances into $\frac{T}{D_T}$ nonoverlapping groups. Let the set of group indices be $G_T = \{1, \frac{T}{D_T}\}$.
2) Let $D_L$ a positive integer with $D_L \geq 2$ and $\frac{L}{D_L}$ is an integer. Segment the $L$ sniffers into $\frac{L}{D_L}$ nonoverlapping groups. Let the set of group indices be $G_L = \{1, \frac{L}{D_L}\}$.
3) For $g_T = 1$ and $g_L = 1$, solve the optimization problem in (8) for all possible associations and select the associations corresponding to $S$ smallest objective function values at the convergence. Record the $D_T$ target and $D_L$ sniffer locations found for all $S$ associations selected.
4) For $1 < g_T \leq \frac{T}{D_T}$ and $g_L = 1$, for each possible association, solve (8), using the sniffer locations found in Step 3 as prior knowledge. For each sniffer set out of $S$ sets found in Step 3, find the association rule that generates the minimum objective function value. Record the $S$ sets of locations of the target.
5) For $g_T = 1$ and $1 < g_L \leq \frac{L}{D_L}$, for each possible association, find the sniffer locations by solving (8), using the target locations found in Step 3 as prior knowledge. For each target set found in Step 3, find the association rule that generates the minimum objective function value.
6) Repeat step 5 for all $g_T \in \left[2, \frac{T}{D_T}\right]$. 

Output: Multipath associations for all the target-sniffer pairs for all symmetry orientations $A_{ij}^{m}(r_1)$, $j \in [1, L]$, $t \in [1, T]$, $s \in [1, S]$, and $l \in [1, P_j(t)]$.

are formed such that Sniffer 1 is a member of all the groups. The target groups are formed similar to Algorithm 1. In this case the optimization in (9) shown at the bottom of previous page, is carried out for all $g_T$ and $g_L$ pairs separately, using $r_1$ as prior knowledge. Unlike Algorithm 1, the output of Algorithm 2 is a unique association, irrespective of the number of symmetry axes in the wall layout.

It should be noted that (8) and (9) are nonconvex optimization problems. Therefore, the optimization algorithm in each case may converge to a local minimum which is different from the global minimum. To tackle this issue, multiple random initializations are used. Furthermore, we use $D_T = D_L = 2$ during the implementation of Algorithms 1 and 2 since it provides a compromise between the accuracy and the computational complexity. If the product $D_T D_L < 4$, the accuracy of optimization result degrades since the number of multipath TDoA observations is insufficient. If $D_T D_L > 4$, the computational complexity of exhaustive search within groups increases such that the execution of the algorithm in real-time is difficult. For $D_T = D_L = 2$, the number of measurement time instances $T$ and the number of sniffers with unknown locations are required to be even numbers.

B. Number of Searches Required in the Proposed Multipath Association Algorithm

For the proposed multipath association algorithms, it can be shown that the total number of searches required to obtain all multipath associations is between $\frac{T}{D_T}\frac{L}{D_L}(N_W)^D T$ and $\frac{T}{D_T}\frac{L}{D_L}(N_W)^D L T$. Therefore, the number of required searches scales linearly with the product $L \times T$, which enables implementation of real-time localization systems. As a numerical example, using 6 sniffers for listening in 20 measurement time instances in an environment with three reflective walls, the number of searches required to obtain all the multipath associations using exhaustive search is between $3^{20}$ and $6^{20}$. For the same setup, when the proposed multipath association algorithm is used with $D_T = D_L = 2$, the total number of required searches reduces to a value between $30(3^4)$ and $30(6^4)$, which enables implementing real-time localization.

V. PROPOSED LOCALIZATION ALGORITHMS

In this section, we present the proposed localization algorithms. We propose two localization algorithms to solve the joint
Algorithm 3: Batch Processing Localization Algorithm.

Input:
- Multipath TDoA estimates for $L$ sniffer obtained at $T$ measurement time instances.
- Multipath associations for all the target-sniffer pairs for all symmetry orientations $A^j_s(t), j \in [1, L], t \in [1, T], s \in [1, S]$, and $l \in [1, P_j(t)]$ obtained using Algorithm 1.

Algorithm Steps:
- If there are no anchor nodes, solve (6) for all symmetry orientations separately. If there is one sniffer with known location, use it in (6) as a known constant.
- If there is one sniffer node with known location, the output is the unique set of estimates of target locations for all measurement time instances and all the sniffer locations.

Output:
- If there are no anchor nodes, the output is the $S$ sets of estimates of target locations for all measurement time instances and all the sniffer locations.

tracking and localization problem using only the estimates of multipath TDoA.

A. Batch Processing Algorithm

In the batch processing algorithm, all the locations of the target and the receivers are found by solving a single optimization problem, which is given in (6). With knowledge of multipath associations is provided by Algorithm 1, it is straightforward to solve (6). Algorithm 3 shows the batch processing localization algorithm. The optimization problem (6) is also nonconvex and multiple random initialization are used when solving. Note that the batch processing algorithm is required to be executed only once after all the measurements of multipath TDoA are acquired. We use the interior point method through the ‘fmincon’ function in MATLAB for constrained optimization to solve (6), (8), and (9).

B. Online Processing Algorithm

In the online processing algorithm, we solve the joint tracking and localization problem using particle filtering [22]. The localization algorithm should be able to find $p(x_M(1 : t), x_R|y(1 : t))$, where $x_M(1 : t)$ is the estimates of all target states from time 1 to $t$ and $y(1 : t)$ is the history of measurements from time 1 to $t$. Note that the target state include both the target location and step size at a given time instance. For particle filtering, some prior knowledge of the target motion is required.

C. Target Motion Model

Let $A \in \mathbb{R}^2$ be the set of all points located inside the room in which the target is moving. Let $\hat{l}(t) = l(t - 1) + d(t)$, where $l(t - 1)$ is the target location at measurement time instance $t - 1$ and $d(t) \in \mathbb{R}^2$ is the step taken by the target if it moves from $l(t - 1)$ to $\hat{l}(t)$. Note that $\hat{l}(t)$ may be a point located outside the room. Let $\bar{d}(t) = d(t - 1) + n(t)$, where $d(t - 1) \in \mathbb{R}^2$ is the step taken by the target during the time interval $[t - 2, t - 1]$ and $n(t) \in \mathbb{R}^2$ is a zero-mean Gaussian vector with an unknown covariance matrix $C_M$. From (3), the reflection of $l(t)$ off the wall $W_k$ is $\hat{\alpha}_k(t) = l(t) - \frac{2(w_k^T l(t) + b_k)}{|w_k|^2}w_k$, where wall $W_k$ is the wall closest to $l(t - 1)$.

The discrete-time motion model of the target is summarized in (10) shown at the bottom of this page. If the target is close to a wall such that $l(t)$ is outside the room and $\hat{\alpha}_k(t)$ is inside the room, the model reflects it back inside by setting $l(t) = \hat{\alpha}_k(t)$, to capture the behavior of a person walking close to the wall. This is given by Case 2 of (10). If the target is close to a corner where two walls $W_k$ and $W_q$ intersect, such that both $l(t)$ and $\hat{\alpha}_k(t)$ are outside the room, it reflects off wall $W_k$ and then off wall $W_q$ by setting $l(t) = \tilde{\alpha}_k(t) - \frac{2(w_q^T \tilde{\alpha}_k(t) + b_q)}{|w_q|^2}w_q$, which is given by Case 3 of (10). Here, Wall $W_k$ is closer to the target location $l(t - 1)$ than $W_q$. In Fig. 1, target locations A and B are examples which correspond to Cases 2 and 3 of (10), respectively. The state of the mobile target is expressed as

$$x_M(t) = [l(t)^T, d(t)^T]^T$$

and the state vector of the localization problem as

$$x(t) = [x_M(t)^T, x_R(t)^T]^T$$

where $x_R = [x_R^T, \ldots, x_R^T]^T$.

The online processing algorithm consists of two main steps [23]. In the time update step, $p(x_M(1 : t), x_R|y(1 : t - 1))$ is estimated using the assumed target motion model and the distribution $p(x_M(t - 1), x_R|y(1 : t - 1))$. In the measurement update step, $p(x_M(t), x_R|y(1 : t))$ is found using the new measurement $y(t)$.

The measurement model given in (5) is nonlinear. Therefore, we implement a hierarchical particle filtering algorithm similar to [18]. The particle filter consists of two levels with the top-level particle filter which consists of particles of target locations. Each target particle has $L$ subspace filters, which correspond to the static sniffer. The posterior distribution function of $x_M(t)$ conditioned on $y(1 : t)$ is approximated by

$$p(x_M(t)|y(1 : t)) \approx \sum_{i=1}^{N_T} w_M(t) \delta(x_M(t) - x_M(t)_i)$$

$$l(t) = \begin{cases} l(t - 1) + d(t) & \text{if } \hat{l}(t) = l(t - 1) + d(t) + n(t) \in A \\ \alpha_k(t) = \hat{l}(t) - \frac{2(w_k^T l(t) + b_k)}{|w_k|^2}w_k \text{ and } d(t) = \alpha_k(t) - l(t - 1) & \text{if } \hat{l}(t) \notin A \text{ and } \alpha_k(t) \in A \\ \tilde{\alpha}_k(t) = \alpha_k(t) - \frac{2(w_q^T \tilde{\alpha}_k(t) + b_q)}{|w_q|^2}w_q \text{ and } d(t) = \tilde{\alpha}_k(t) - l(t - 1) & \text{if } \hat{l}(t) \notin A \text{ and } \tilde{\alpha}_k(t) \notin A \end{cases}$$

Authorized licensed use limited to: CSIRO Information Technology Services. Downloaded on February 12,2020 at 00:46:52 UTC from IEEE Xplore. Restrictions apply.
where \( N_T \) is the number of target particles and \( w_{M,i} \) is the weight of the \( i \)th particle at measurement time instance \( t \).

The weights of the top-level particle filter can be expressed as

\[
w_{M,i}(t) \propto w_{M,i}(t-1)p(Y(t)|x_{M,i}, Y(t-1))
\]

\[
= w_{M,i}(t-1) \int p(Y(t)|x_{M,i}, X_R, Y(t-1))
\]

\[
\times p(X_R|x_{M,i}, Y(t-1)) dX_R.
\]

(14)

\( Y(t) \) is conditionally independent on \( Y(t-1) \) given \( x_{M,i} \) and \( X_R \). Furthermore, the observations of each sniffer at time \( t \) are conditionally independent given \( x_{M,i} \) and sniffer locations. Moreover, the observation of each sniffer depends only on the corresponding sniffer location, given the target particles. Therefore, \( I_1 \) in (14) can be expressed as

\[
I_1 = p(Y(t)|x_{M,i}, X_R)
\]

\[
= \prod_{j=1}^{L} p(y_j(t)|x_{M,i}, X_R) = \prod_{j=1}^{L} p(y_j(t)|x_{M,i}, r_j).
\]

(15)

Recall that all the sniffers remain stationary. It is assumed that the sniffers are deployed independently of each other, i.e., \( p(X_R) = \prod_{j=1}^{L} p(r_j) \). Based on this assumption, \( I_2 \) in (14) is approximated as

\[
I_2 \approx \prod_{j=1}^{L} \int p(r_j|x_{M,i}, Y(t-1))
\]

\[
= \prod_{j=1}^{L} p(r_j|x_{M,i}, y_j(t-1)).
\]

(16)

where the equality in (16) is due to the fact that the each sniffer location \( r_j \) depends only on the observations of given sniffer, given the target particle locations.

From (15) and (16),

\[
w_{M,i}(t) \approx w_{M,i}(t-1) \prod_{j=1}^{L} p(y_j(t)|x_{M,i}, r_j)
\]

\[
\times p(r_j|x_{M,i}, y_j(t-1)) dr_j.
\]

(17)

The conditional distribution \( p(r_j|x_{M,i}, y_j(t-1)) \) is approximated by

\[
p(r_j|x_{M,i}, y_j(t-1)) \approx \sum_{p=1}^{N_{R_j}} w_{R_{j,p},i}^{p}(t) \delta(r_j - r_{j,i}^{p})
\]

(18)

where \( r_{j,i}^{p} \) are the sniffer location particles correspond to \( i \)th target particle and \( j \)th sniffer, \( w_{R_{j,p},i}^{p}(t) \) are the weights of \( r_{j,i}^{p} \), and \( N_{R_j} \) is the number of particles used for the sniffer \( R_j \). The sniffer weights \( w_{R_{j,i},i}^{p}(t) \) are found by using (5) as

\[
w_{R_{j,i},i}^{p}(t) \propto w_{R_{j,i},i}^{p}(t-1)p(y_j(t)|x_{M,i}, r_{j,i}^{p}).
\]

(19)

Note that the particle filters correspond to the sniffer locations can be executed in parallel due to (17). Therefore, the complexity of the proposed particle filtering algorithm scales linearly with the number of sniffer nodes.

Using (17), (18), and (19), the weights \( w_{M,i}(t) \) are found by

\[
w_{M,i}(t) \propto w_{M,i}(t-1) \prod_{j=1}^{L} \sum_{p=1}^{N_{R_j}} w_{R_{j,i},i}^{p}(t-1)p(y_j(t)|x_{M,i}, r_{j,i}^{p}).
\]

(20)

Algorithm 4 presents the pseudocode of the online processing localization algorithm, which is based on (20). At \( t = 1 \), we generate \( N_{T_1} \) target location particles distributed uniformly in the room area, which are denoted by \( \{l(1), i[1, N_{T_1}]\} \). To initialize the target step, a two-dimensional grid with \( N_d \) grid points is used. \( d_x \) and \( d_y \) are the width and the height of this grid, respectively. The target state particles at \( t = 1 \) is

\[
\{x_{M}(1), i[1, N_{T_1}, N_d] = \{l(1), a, d(1), b\} a[1, N_{T_1}, b[1, N_d] \}
\]

(21)

where \( \{l(1), i[1, N_{T_1}]\} \) are the particles filtered during the initialization phase of Algorithm 4. Hence, the number of target particles \( N_T = N_{T_1} N_d \). The time-updated target particles are obtained using the motion model in (10). Using Algorithm 4, the minimum mean square error (MMSE) estimate of the target state at time \( t \) is

\[
\hat{x}_{M}(t) = \sum_{i=1}^{N_T} w_{M,i}(t)x_{M,i}(t)
\]

(22)

and the estimated location of sniffer \( j \) is

\[
\hat{r}_j = \sum_{i=1}^{N_T} \sum_{p=1}^{N_{R_j}} w_{M,i}^{p}(t)w_{R_{j,i},i}^{p}(t)r_{j,i}^{p}(t).
\]

(23)

Note that the location estimations given in (22) and (23) are for a single orientation of symmetry. The estimations of target and sniffer locations for the remaining symmetry orientations are obtained similarly applying the corresponding multipath associations in Algorithm 4. If \( r_1 \) is known, a unique set of estimations for the target and sniffer locations are found by Algorithm 4, using \( r_1 \) as prior knowledge.

VI. NUMERICAL RESULTS

In this section, the performance of the proposed passive tracking system is verified using simulations. We use the room layout given in Fig. 1. The sniffers are located outside the building. For the online processing algorithm, we use \( N_T = 5000 \) and \( N_{R_j} = 200 \) for all the sniffers. In the simulations true covariance matrix of \( n(t) \) in (10) of the target motion is 0.3I. Recall that this value is not known by the localization algorithm. Instead, we use a realistic assumption of \( I \) for the covariance matrix of \( n(t) \) in the online processing algorithm. For multipath association algorithms 1 and 2, we use \( D_T = D_L = 2 \). The SNR is 10 dB. \( N_{d} \) is the number of grid points used generate the initial steps \( d(1) = 50 \) with \( d_x = 1 \) m and \( d_y = 2 \) m.

Fig. 4 shows the empirical cumulative distribution function (CDF) of the target localization error obtained for the two proposed localization algorithms. The blue curves show the localization error results obtained with one anchor node and
in Section III-D, there are two equivalent estimates of target and each with 20 measurement time instances, are used. As discussed empirical CDFs, 300 random trajectories inside the indoor area, sniffer locations are known, for comparison. To generate the red curves depict the results obtained without any anchor nodes. The green curve shows the results obtained with all the sniffer locations are known, for comparison. To generate the empirical CDFs, 300 random trajectories inside the indoor area, each with 20 measurement time instances, are used. As discussed in Section III-D, there are two equivalent estimates of target and sniffer locations due the symmetry of the wall layout when no anchor node is used. One equivalent set of sniffer and target location estimates is the mirror image of the other by rotating the set around the axis of symmetry by 180°. Fig. 4 shows the errors for the target location estimates from the equivalent set that correspond to the true target and sniffer locations.

Fig. 4 shows that for SNR 10 dB, both the proposed localization algorithms achieve a localization accuracy of 40 cm with probability of 0.95 without using any anchor node. The accuracy of localization improves when one anchor node is used since having an anchor nodes reduces the number of variables to be found by one, which improves the accuracy of multipath association algorithm. From Algorithm 4, the online processing algorithm uses partial knowledge of target motion, which improves its accuracy compared to the batch processing algorithm (Algorithm 3) which does not use any knowledge of target motion.

In Fig. 5, the error CDFs of target localization obtained using the multipath associations provided by the proposed multipath association algorithms are compared with the case where localization algorithm is executed with ground truth of all

**Algorithm 4: Online Processing Localization Algorithm.**

**Input:**
- Measurement matrix $Y(1 : t)$, $\sigma^2$, Locations of the walls
- Multipath associations for all the target-sniffer pairs for a given symmetry orientation $A_{j,t}^{L,s}$, $j \in [1, L]$, $t \in [1, T]$, and $l \in [1, P_j(t)]$ obtained using Algorithm 1.

**Initialization** ($t = 1$):
1. Generate $N_{T_i}$ particles of the target $\{I_{(1)}\}_{i \in [1, N_{T_i}]}$.
2. For each target particle $I_{(1)}$, generate $N_{R_j}$ particles of $r_j, \{r_{j,i}^p(1)\}_{j \in [1, L], p \in [1, N_{R_j}]}$.
3. Obtain the sniffer particle weights for each sniffer $w_{p,j,i}^R(1) = \frac{p(Y(1) | I_{(1)}, r_{j,i}^p(1))}{\sum_{p=1}^{N_{R_j}} w_{p,j,i}^R(1)}$, which can be obtained from the assumed target motion model in (10), with the covariance matrix of $\mathbf{n}(t)$ is arbitrarily selected.

**Localization for** $t > 1$:
1. Obtain $X_M(t)$ using $p(X(t)|X(t-1))$, which can be obtained from the assumed target motion model in (10), with the covariance matrix of $\mathbf{n}(t)$ is arbitrarily selected.
2. Obtain the sniffer particles at time $t$, $r_{j,i}^p(t) = r_{j,i}^p(t-1)$
3. Obtain the sniffer particle weights for each sniffer $w_{p,j,i}(t) = \frac{p(Y(t) | X_M(t), r_{j,i}^p(t))}{\sum_{p=1}^{N_{R_j}} w_{p,j,i}^R(t)}$, which improves its accuracy compared to the batch processing algorithm (Algorithm 3) which does not use any anchor node. The accuracy of localization improves when one anchor node is used since having an anchor nodes reduces the number of variables to be found by one, which improves the accuracy of multipath association algorithm. From Algorithm 4, the online processing algorithm uses partial knowledge of target motion, which improves its accuracy compared to the batch processing algorithm (Algorithm 3) which does not use any knowledge of target motion.

In Fig. 5, the error CDFs of target localization obtained using the multipath associations provided by the proposed multipath association algorithms are compared with the case where localization algorithm is executed with ground truth of all
multipath associations is known. We use batch processing algorithm (Algorithm 3) for localization.

With perfect knowledge of multipath associations, the target localization error CDF curves for the cases with one anchor is used and no anchor used almost coincide. This is due to the fact that Algorithm 3 carries out the optimization over a large number of variables (20 target locations and 6 sniffer locations). In this case, having prior knowledge of the location of only one sniffer node does not improve the localization accuracy significantly. However, to determine the multipath associations using Algorithms 1 and 2, the optimization is carried out only over two target locations and two sniffer locations at a time. In this case, having the knowledge of one sniffer node can significantly improve the accuracy of determining multipath association, which is pivotal for achieving accurate localization results. This explains the improvement in target localization accuracy achieved with association Algorithm 2, compared to association Algorithm 1.

In Fig 6, the CDFs of localization error obtained by the proposed algorithms for Sniffer 2 and Sniffer 6 are shown. The multipath association Algorithm 1 was used. It is evident that both localization algorithms achieve accuracies of 0.3 and 0.4 m with probability higher than 0.95 for the locations of Sniffer 2 and Sniffer 6, respectively.

In Fig. 7, we investigate the impact of the number of sniffers used and their locations of deployment on the target localization error. We consider the scenarios where the the sniffers are randomly deployed along Wall 4 in Fig. 1 and sniffer are deployed randomly in a 5 × 15 m area outside Wall 4. All the curves were generated using Algorithm 1 as the multipath association algorithm and batch processing (Algorithm 3) localization algorithm. Fig. 7 shows that the target localization accuracy achieved by deploying sniffers along the outer wall is higher than the localization accuracy achieved by deploying sniffers in the outer area, which implies that the deployment of sniffers closer to the target enhances the target localization accuracy. However, this performance difference degrades as the number of deployed sniffers increases. Furthermore, the performance advantage achieved by increasing the deployed number of sniffers diminishes as the number of sniffer nodes increases.

VII. EXPERIMENTAL RESULTS

In this section, we present the performance results of the proposed passive tracking system obtained from experiments conducted in an anechoic chamber using hardware implementation in the 5.745 GHz frequency band. We used one moving target and 6 passive sniffers. Due to practical limitations in the anechoic chamber, an environment with single reflective wall was used. The experiment setup is given in Fig. 8. Similar to the simulations, we use $N_T = 5000$ and $N_R_j = 200$ for all the sniffers.

We designed and developed the WiFi-WASP sniffers, which listen to IEEE 802.11ac channels, to collect CSI. Fig. 9 shows the block diagram and photo of a sniffer node, which is built from commercial MicroZed development boards and custom-designed AD9361 [24] software defined radio (SDR) modules. Since the bandwidth of each SDR module is limited to 56 MHz, two SDR modules are tuned to overlapping frequency bands to cover a 80 MHz WiFi channel. The sniffer monitors the WiFi channel passively during operation and captures raw I/Q samples of WiFi packets for processing. Particularly, the CSI was estimated using the long Very High Throughput Long Training Field (VHT-LTF) in each 802.11ac packet.

We collected CSI of subcarriers from about 200 packets at each sniffer for a given target location and estimated multipath TDoA by applying the MUSIC algorithm for each packet. The estimated TDoA for at a given sniffer for a target location is the average of TDoA estimated for each packet. For measurement time instance 4, the distance differences which correspond to the estimated TDoA are shown as histograms in Fig. 10 for Rx 1, Rx 4, and Rx 5. It can be seen that the MUSIC algorithm generates accurate TDoA values for a signal bandwidth of 80 MHz.
Fig. 9. (a) Block diagram of the WiFi packet sniffer used in the experiments. (b) A photo of the sniffer, which shows the Microzed, carrier board, and SDR modules sharing a single antenna.

Fig. 10. Results of estimation of TDoA using the MUSIC Algorithm for measurement time instance 4. (a) sniffer 1 (b) sniffer 4 (c) sniffer 5.

Fig. 11. Estimated target and sniffer locations obtained over a single realization of the performance demonstration of the localization algorithm in the anechoic chamber.

Fig. 11 graphically shows the true target locations and the target location estimates obtained by applying the batch processing algorithm (Algorithm 3) and the online processing algorithm (Algorithm 4) on the experiment data collected over a single experiment. The estimated sniffer locations are not shown here for simplicity. The location of Sniffer 1 is assumed to be known. The initial target particles are randomly drawn from the 2 × 2 m area shown in Fig. 11 for the online processing algorithm. For the batch processing algorithm, 50 random initial location configurations are used.

It can be observed that the proposed passive localization algorithms are capable of accurately estimating the target and sniffer locations. In Table I, the target localization error results of the two proposed algorithms are compared. The online processing algorithm shows an excellent target localization accuracy with a maximum localization error of 33 cm. The batch processing algorithm is less accurate. This behavior is consistent with the simulation results obtained using an environment with three walls in Fig. 4. The localization errors of each sniffer node are compared in Table II. Both proposed algorithms achieve submeter localization errors for all sniffer locations.

As can be seen from Table I, the average localization accuracy of our proposed online processing algorithm is 20 cm, and it is 50 cm for the our proposed batch processing algorithm. These figures are significantly better than WiFiSLAM [3] and WiFi GraphSLAM [4] approaches, of which the reported average localization accuracies were 3.97 and 2.23 m, respectively. As pointed out previously, this demonstrates the benefits of our approach which exploits multipath propagation.

Our algorithms can be executed in real time with computational complexity comparable to WiFiSLAM [3] and WiFi GraphSLAM [4] algorithms. Although our proposed online processing algorithm is required to search through the particle space of all sniffers, we perform this search in parallel, as in (17), which reduces the computational complexity from a potential \(O(TL^2)\), to a practical search with \(O(TL)\). The computational complexity of the proposed batch processing algorithm scales polynomially
with the number of target locations and sniffer nodes, since the asymptotic computational complexity of the optimization operation is approximately $O(TL(T + L)^2)$. The asymptotic computational complexities of WiFiSLAM and WiFi Graph-SLAM systems are $O(T^3)$ [3] and $O(T^2)$ [4], respectively. As a numerical example, with $N_T = 5000$ and $N_{R} = 200$, the Online Processing algorithm computes each target location in approximately 25 s, using parallel processing in MATLAB on a standard computer which has a quadcore processor with clock rate of 3.50 GHz. Our proposed batch processing algorithm completes its execution for all eight target locations in the experiment setup in approximately 12 s with 50 random initializations. Of course, these execution times would be orders of magnitude smaller if implemented in a field-programmable gate array or an application-specific integrated circuit.

### VIII. CONCLUSION

In this article, a novel passive tracking system was developed using IEEE 802.11ac signals, which uses only the estimates of multipath TDoA to jointly track a noncooperating moving target and localize the sniffers. The proposed system did not use any synchronization and we did not assume any prior knowledge of sniffer locations. A novel multipath association algorithm, which has a computational complexity scales linearly with the number of target locations and sniffers was proposed. Two localization algorithms were developed, batch and online processing. Using experiments, it was verified that the online processing algorithm achieves an accuracy better than 30 cm for all the target locations in an environment layout with one wall.

### ACKNOWLEDGMENT

The authors would like to thank Julian Sorensen, Adrian Donarski and John Kitchen of Defence Science and Technology Group, Australia, for the valuable support provided.

### TABLE I

<table>
<thead>
<tr>
<th>Measurement time instance</th>
<th>Localization error (m) - Batch Processing (Algorithm 3)</th>
<th>Localization error (m) - Online Processing (Algorithm 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5382</td>
<td>0.1604</td>
</tr>
<tr>
<td>2</td>
<td>1.201</td>
<td>0.97</td>
</tr>
<tr>
<td>3</td>
<td>1.1733</td>
<td>0.1304</td>
</tr>
<tr>
<td>4</td>
<td>0.4905</td>
<td>0.3341</td>
</tr>
<tr>
<td>5</td>
<td>0.1328</td>
<td>0.2247</td>
</tr>
<tr>
<td>6</td>
<td>0.3652</td>
<td>0.1850</td>
</tr>
<tr>
<td>7</td>
<td>0.8669</td>
<td>0.1935</td>
</tr>
<tr>
<td>8</td>
<td>0.3025</td>
<td>0.2174</td>
</tr>
</tbody>
</table>

### TABLE II

<table>
<thead>
<tr>
<th>Sniffer Index</th>
<th>Localization error (m) - Batch Processing (Algorithm 3)</th>
<th>Localization error (m) - Online Processing (Algorithm 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.4486</td>
<td>0.6588</td>
</tr>
<tr>
<td>3</td>
<td>0.3701</td>
<td>0.6650</td>
</tr>
<tr>
<td>4</td>
<td>3002</td>
<td>0.4722</td>
</tr>
<tr>
<td>5</td>
<td>0.5420</td>
<td>0.2703</td>
</tr>
<tr>
<td>6</td>
<td>0.3463</td>
<td>0.2768</td>
</tr>
</tbody>
</table>

### REFERENCES