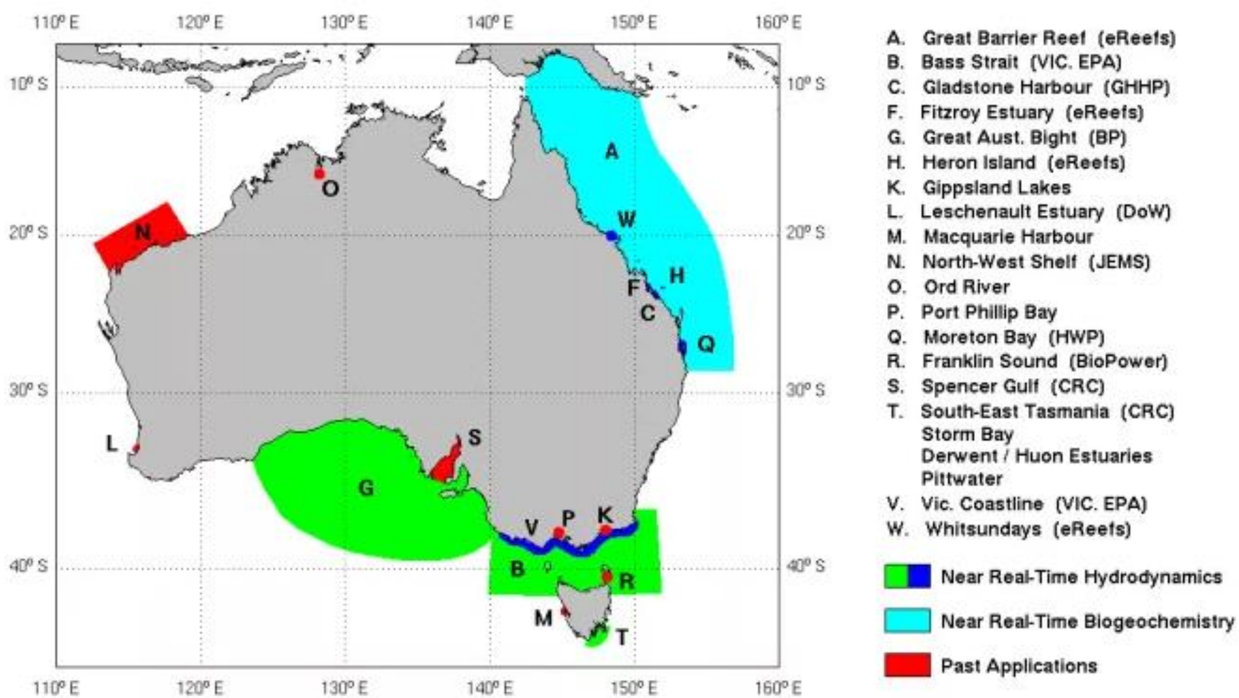
COMPAS – A coastal version of MPAS-O

M. Herzfeld

October 2018

Coastal Modelling around Australia

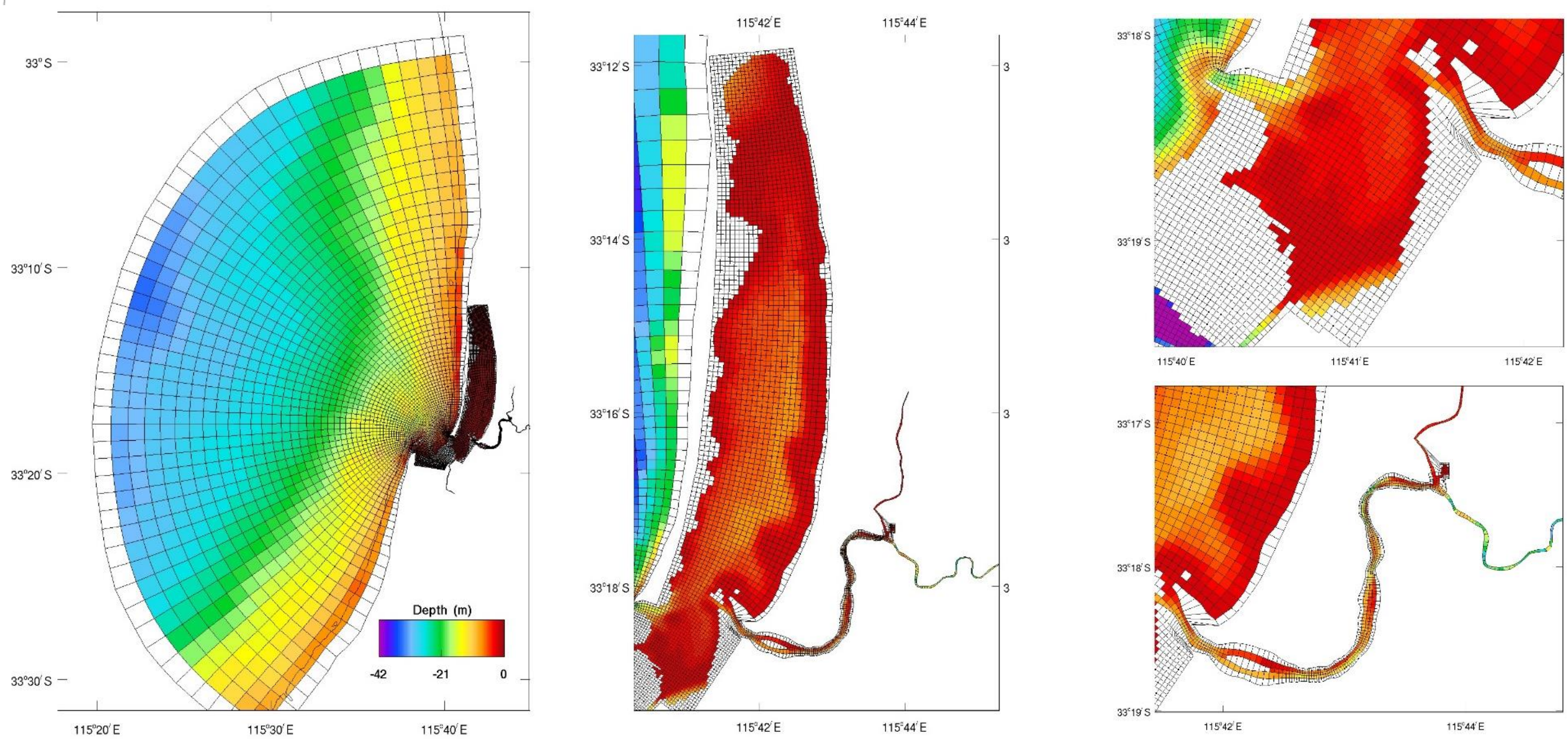
- **Coastal Environmental Modelling team (CEM) has been modeling coastal Australia for > 25 years,**
- **Finite difference structured we have used have served us well.**
  - M3D, MECO, SHOC



Coastal models prioritize resolution / speed

- **Current models use orthogonal curvilinear grids**
- **Underlying code is GRIDGEN (Pavel Sakov)**
- **Coupled to a matlab GUI**
- **Supports complex grids via branching**

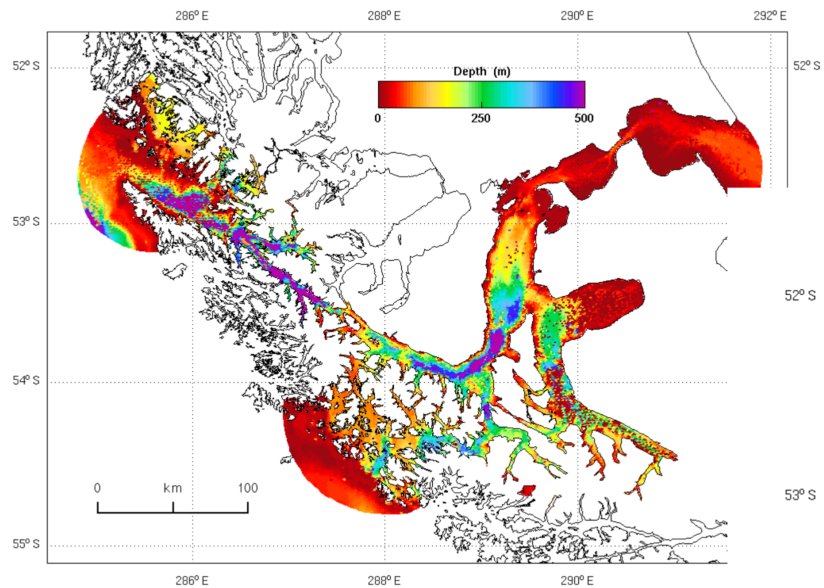
Leschenault Estuary, WA. Built by John Andrewartha

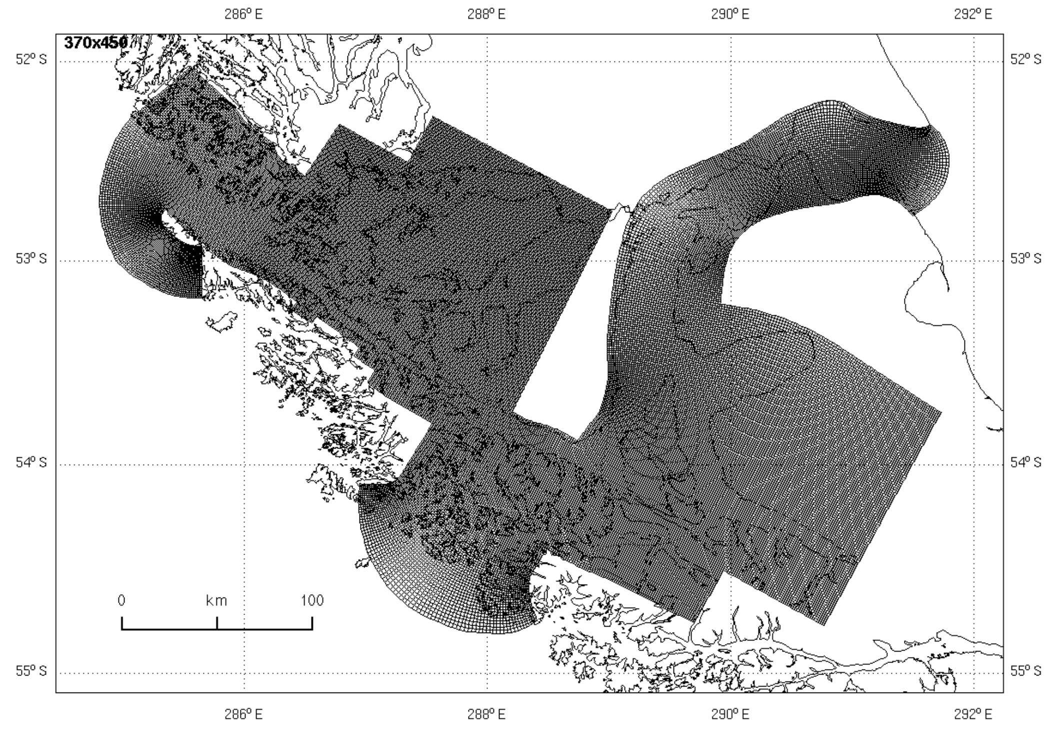


Grid generation in complex geographies

Grid from SHOC

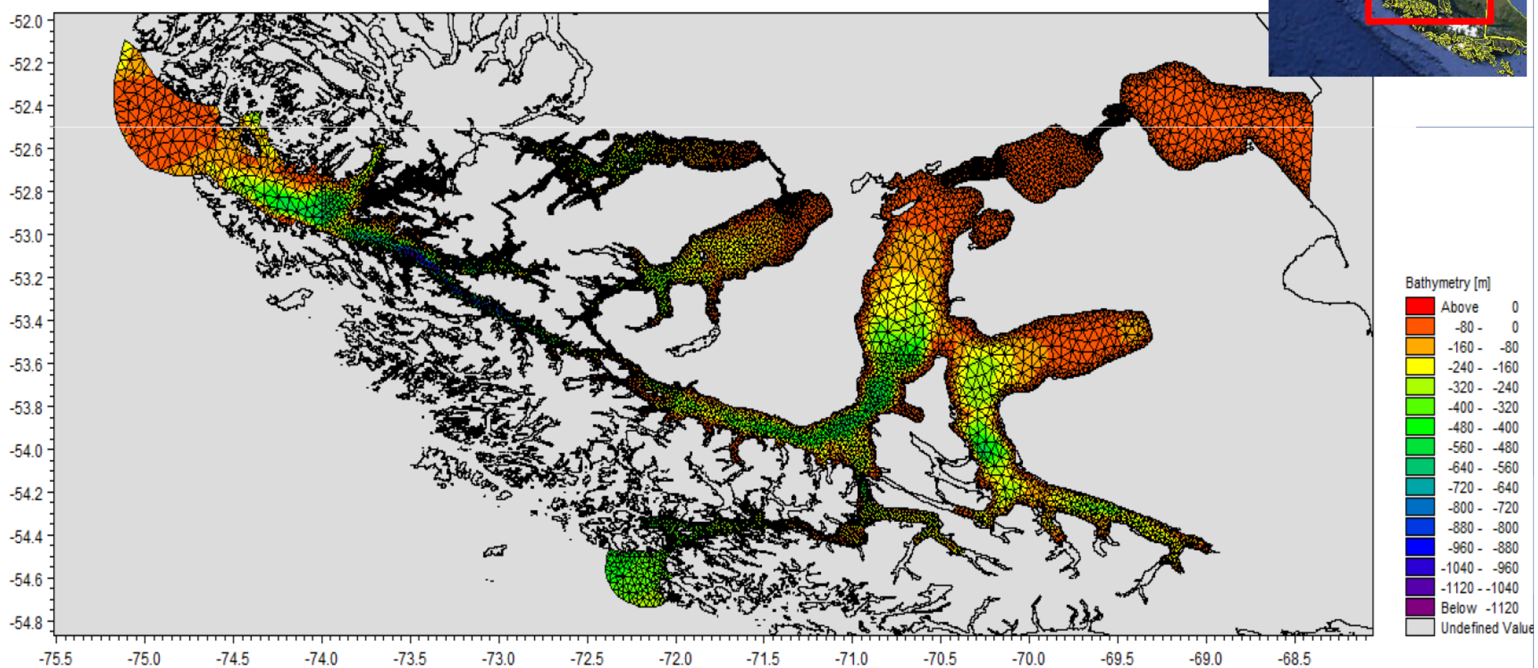
Region XII: Patagonia, Chile



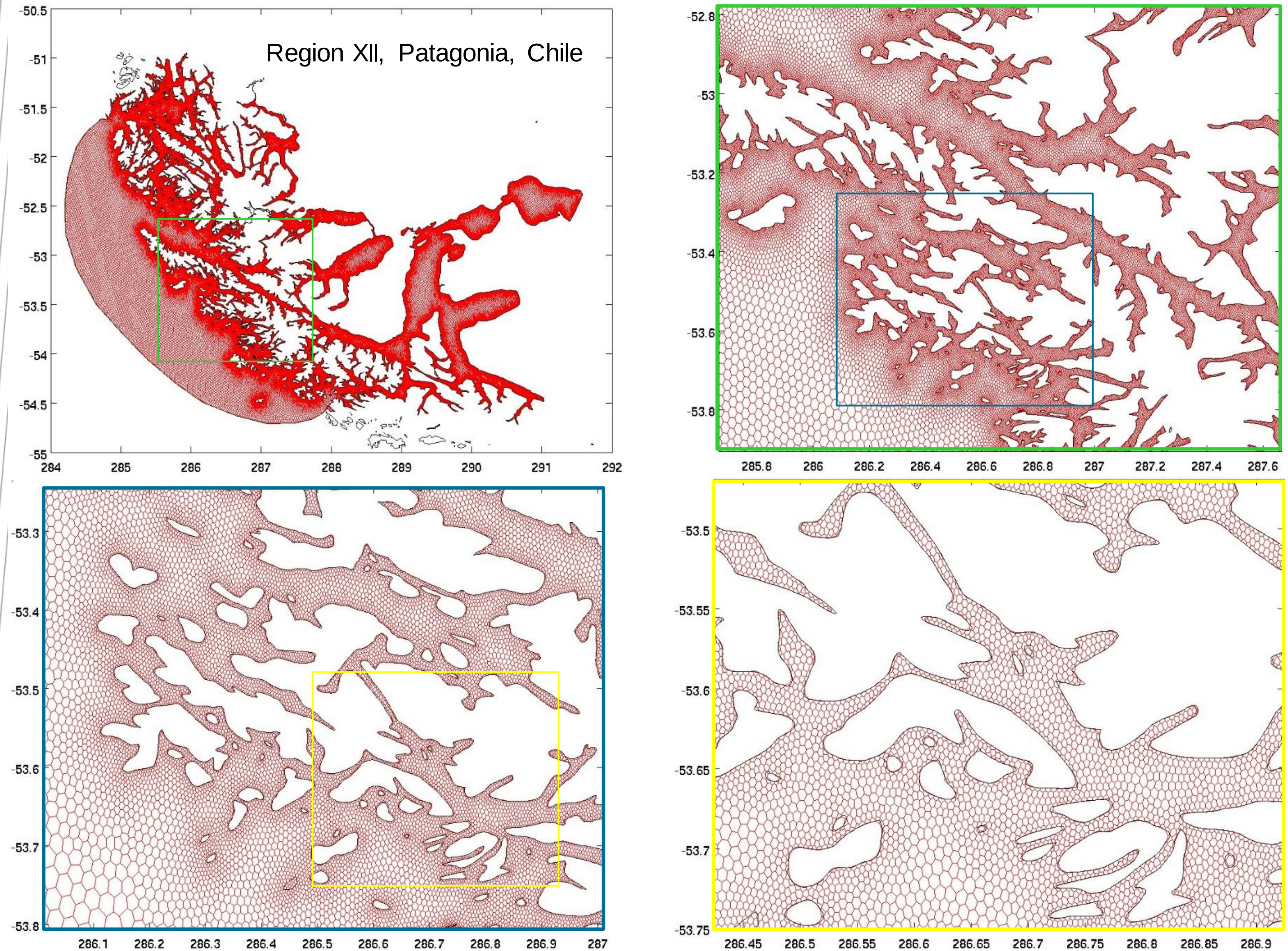


Equivalent unstructured mesh

Mesh from MIKE3D FM
Generated by Elias Pinilla, IFOP, Chile

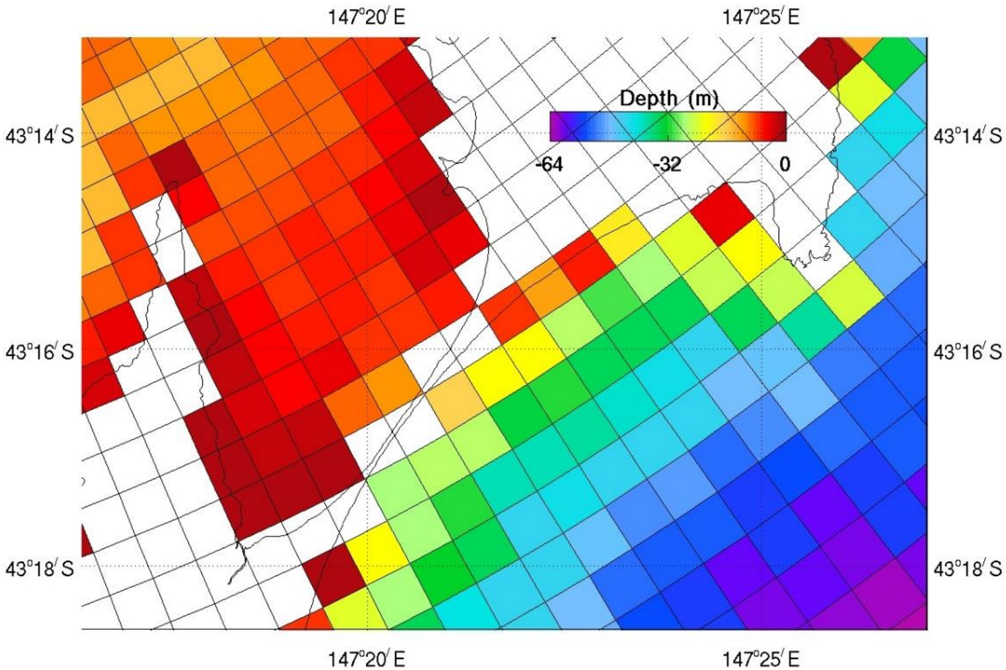


Unstructured models provide superior resolution transition

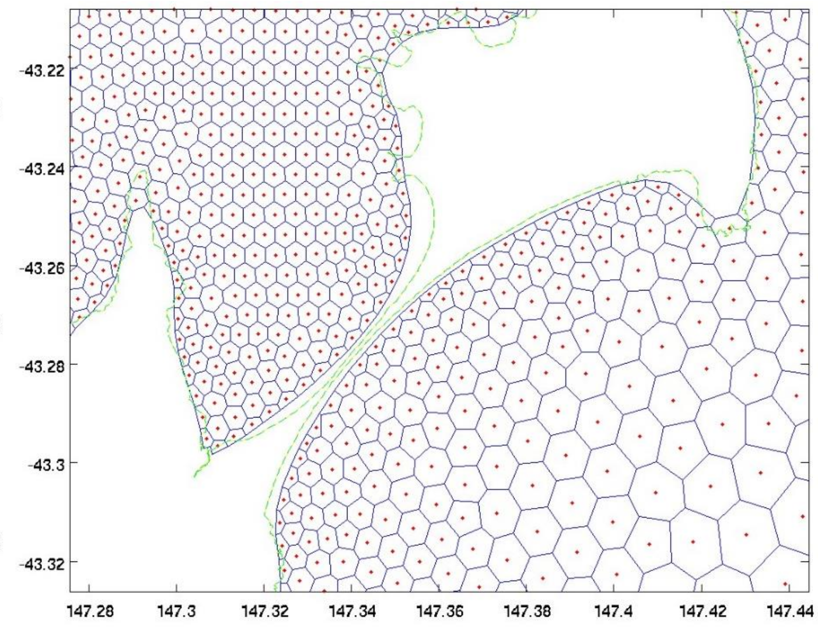


Superior coastline representation

Orthogonal curvilinear



Unstructured



Motivation for unstructured models

- **Current finite difference models are pushing the limits of resolving certain geographies,**
- **Current downscaling approaches are using too many nests (bridging models) to satisfy boundary nesting ratios,**
- **Unstructured approaches offer more flexibility in mesh generation and boundary matching,**
- **International coastal modelling efforts are gravitating towards unstructured approaches.**

Types of models

- **Finite difference**
  - Structured grids
  - Finite difference solution to partial derivatives
    - Taylor series expansions to approximate derivatives
- **Finite element**
  - Unstructured grids
  - Expand fields in basis functions defined on elements and analytically manipulate
  - Continuous or discrete elements
- **Finite volume**
  - Unstructured grids
  - Integrate over control volumes to derive discretized equations

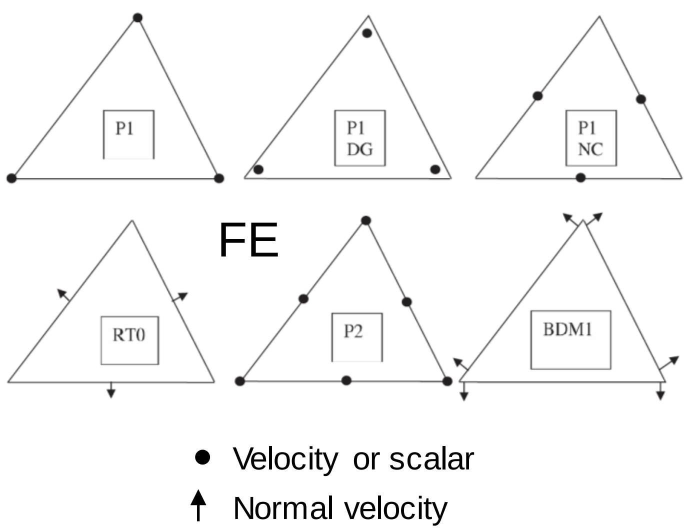
Unstructured models have issues

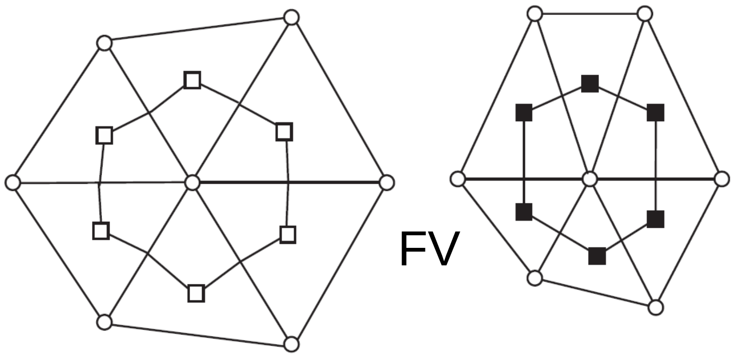
- **Speed: ‘Codes designed to work on unstructured meshes are as a rule slower than their regular mesh counterparts per degree of freedom’ – Danilov (2013),**
- **Generation of spurious modes,**
  - See Hanert (2004) Towards a Finite Element Ocean Circulation Model. Ph.D. thesis.
  - Danilov (2010) for triangular meshes: C-grid
  - Stabilization against pressure modes: A-grids
- **Geostrophy – no stationary geostrophic modes,**
- **Conservation,**
  - Discrete FE only conserves momentum & kinetic energy globally (Peron (2000) Journal of Computational Physics 159, 58–89),
  - Conservation expressed as cluster-weighted form that includes distributed fluxes (Perot (2011) Annu. Rev. Fluid Mech. 43, 299-318).
- **Resolving mesoscale baroclinic instability.**

Danilov, S. (2013) Ocean modelling on unstructured meshes. Ocean Modelling, 69, 195-210.

Unstructured approaches

- Models use a variety of placement of variables:





- No consensus – all have particular problems.

Unstructured modelling packages

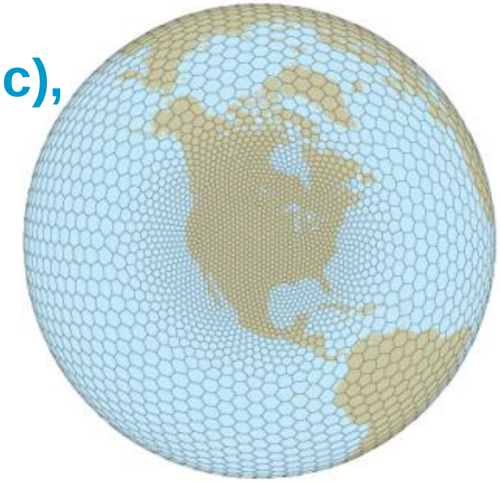
- **Finite Volume (FV)**
  - MPAS (Hex C-grid)
  - FVCOM ($P_0 - P_1$)
  - MIKE FM (cell-cell)
  - SUNTANS (Tri C-grid)
  - ICON (Tri C-grid)
- **Finite element (FE)**
  - ICOM / Fluidity: Continuous ($P_1^{DG} - P_2$)
  - FESOM: Continuous ($P_1 - P_1$)
  - SLIM ($P_1^{nc} - P_1$)
  - SELFE
  - ADCIRC: Continuous ($P_1 - P_1$)
  - TELEMAC
- **Finite volume / element**
  - SCHISM
  - ELCIRC

Choice of models to adopt

- **Essential:**
  - Not finite element
    - continuous FE suboptimal for oceanography - unwanted features in hydrostatic case,
    - discrete are too computationally expensive.
  - Speed comparable to current structures models
  - Not implicit (limits OBC implementation / distributed processing)
  - Minimal stabilization procedures
  - Ease of mesh generation
- **Desirable**
  - Mode split
  - Hydrostatic, Bousenesq
  - C-grid
  - Grid agnostic

Choose to use the MPAS framework

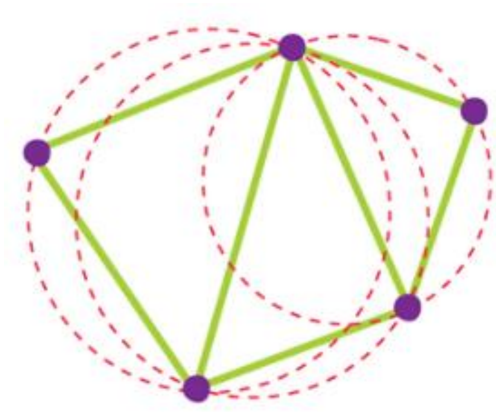
- **MPAS – Model for Prediction Across Scales. Developed by Todd Ringler at Los Alamos (https://mpas-dev.github.io/)**
- **Used for climate prediction modelling,**
- **Contains ocean, atmosphere and sea-ice modules,**
- **ALE vertical coordinate,**
- **Uses Delaunay dual grid; Voronoi (grid agnostic),**
- **MPAS-O* : Ocean model component,**
  - Conserves volume, mass, momentum and vorticity,
  - Supports a stationary geostrophic mode,
  - Generates mesoscale baroclinic instabilities,
  - No stabilization,
  - Uses a C-grid placement of variables,
  - Uses the vector invariant momentum advection approach.



* Ringler, T., Petersen, M., Higdon, R. L., Jacobsen, D., Jones, P. W., & Maltrud, M. (2013). Ocean Modelling. Ocean Modelling, 69(C), 211–232. doi:10.1016/j.ocemod.2013.04.010

Underlying mesh – Delaunay triangulation

- **Delaunay triangulation:** no vertex falls in the interior of the circumcircle (circle that passes through all three vertices) of any triangle in the triangulation



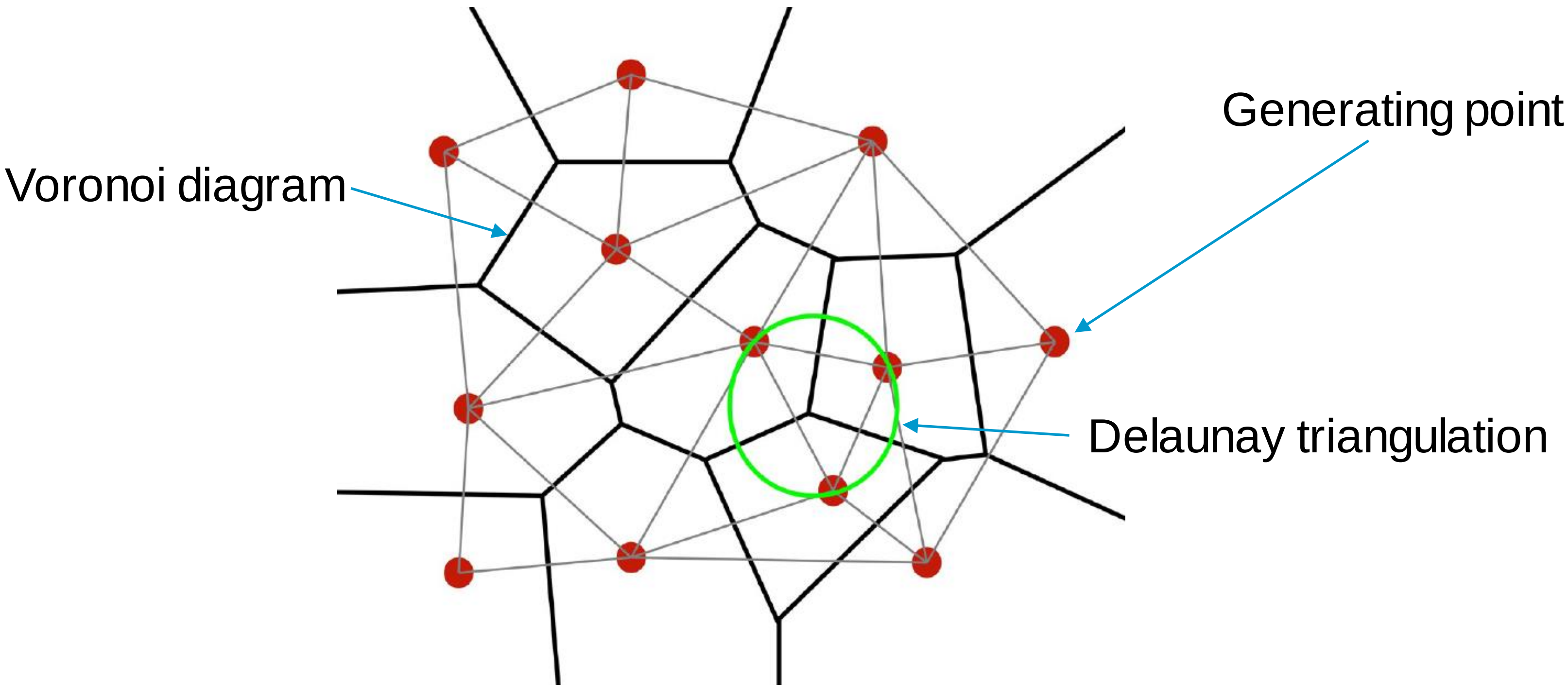
- **Can be generated using existing software, e.g.**

'Triangle'; https://www.cs.cmu.edu/~quake/triangle.html

'Jigsaw'; https://sites.google.com/site/dengwirda/jigsaw

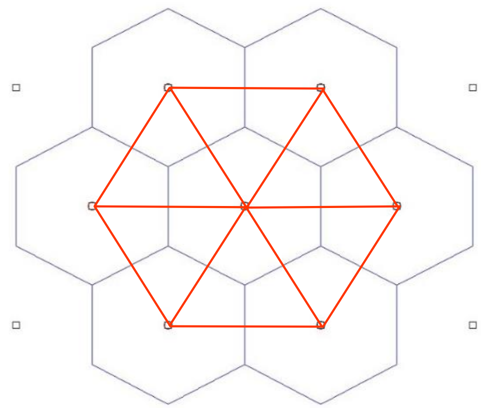
Dual – Voronoi diagram

- **The dual of a Delaunay triangulation is a Voronoi diagram.**
- **Voronoi diagram: all points within a Voronoi cell have a minimum distance to its generating point.**



Centroidal Voronoi Tesselation (CVT)

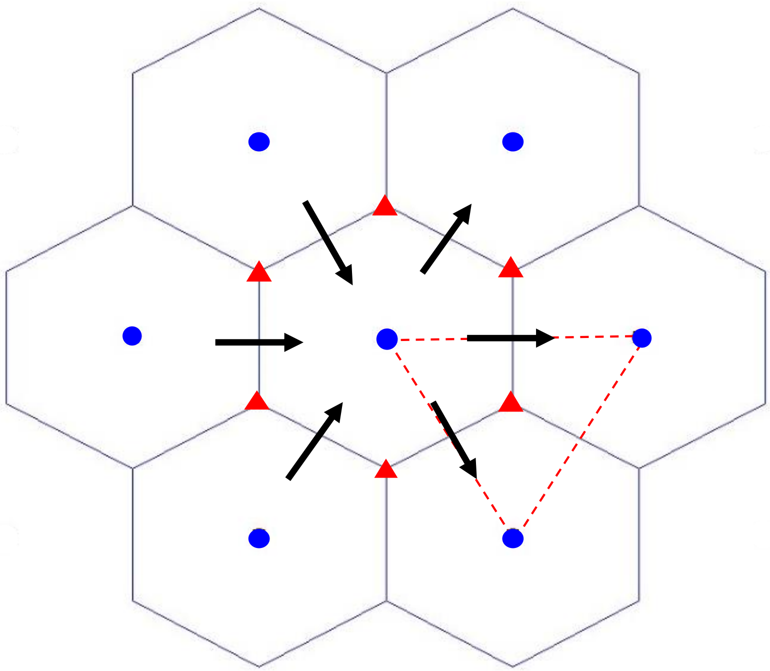
- **Centroidal Voronoi Tesselation (CVT): generating points coincide with the cell's centre of mass.**
  - Generated iteratively using Lloyd's algorithm or k-means clustering.

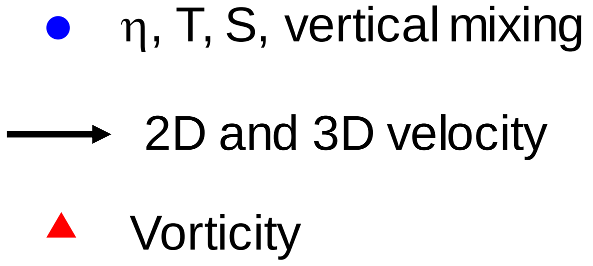


- **MPAS framework uses CVTs and the dual Delaunay triangulation.**
- **The centroids of the mesh must be orthogonal to the edges.**
- **MPAS will operate on any centroidal orthogonal polygon.**

MPAS Placement of variables

- **MPAS uses a C-grid representation for placement of variables;**
  - Sea level / tracers at the polygon centres,
  - Normal velocity on the polygon edges,
  - Vorticity on the vertices.





Algorithm changes for C grid

- **Vertical algorithms remain unchanged (no ALE yet):**
  - Turbulence closure
  - Vertical mixing
  - Implicit vertical momentum advection in vertical mixing
  - Baroclinic pressure
  - Barotropic pressure
- **Need new algorithms for:**
  - Momentum advection
  - Coriolis
  - Horizontal mixing

Momentum advection in MPAS $\frac{Du}{Dt} = 0$

- Flux form (x component)

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = 0$$

- Advective form (x component)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = 0$$

- Vector invariant (vector form, horizontal component)

Tangential velocity

$$\frac{\partial \vec{u}}{\partial t} + \zeta \overbrace{k \times \vec{u}} + \nabla K = 0$$

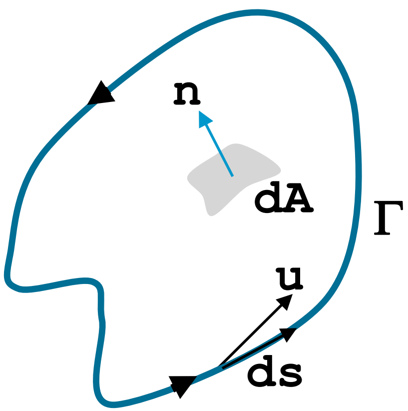
relative vorticity: $\zeta = k \cdot (\nabla \times \vec{u})$

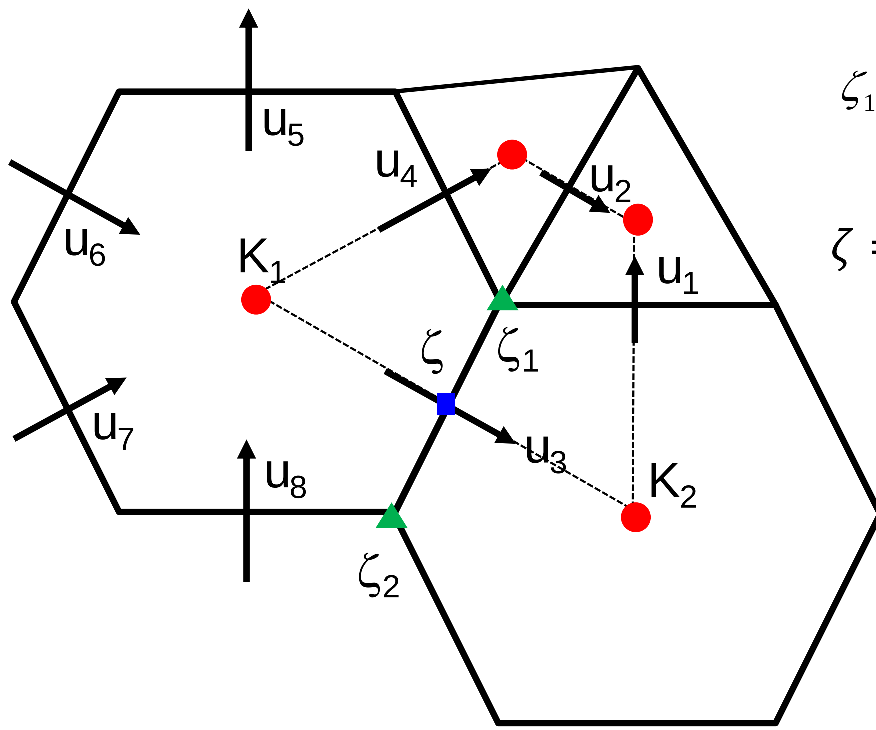
Gradient of kinetic energy K$= \frac{1}{2}|\vec{u}|^2$

Computing K and ζ is easy on arbitrary grids

Vorticity is related to circulation

$$\Gamma = \oint \vec{u}\, ds = \iint \zeta \cdot dA$$





$$K_1 = \frac{1}{2}\sum_{i=3}^{8} u_i^{\,2}$$

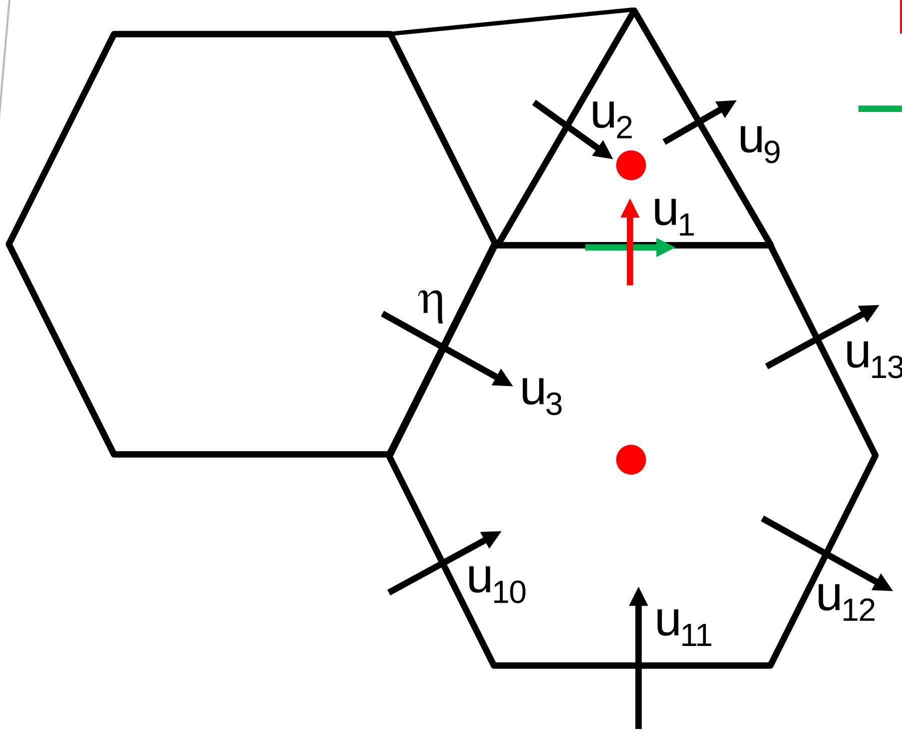
$$\zeta_1 = \frac{1}{A}\sum_{i=1}^{4} u_i dx$$

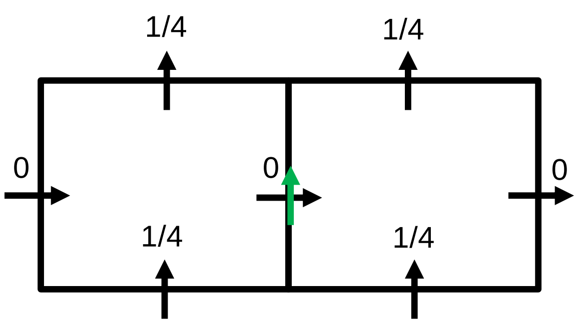
$$\zeta = \frac{1}{2}(\zeta_1 + \zeta_2)$$

Require the tangential velocity also

Normal velocity at edge 1

Tangential velocity at edge 1





Quad grid

Tangential velocities are reconstructed using Thuburn et al (2009), J. Comput. Phys., 228, 8321-8335.

1. Tangential velocity at a face uses all normal velocities from edges associated with the centres that share that edge.
2. Unique weights (w_{en}) are assigned to the above edges.
3. $$u_{et} = \sum_n w_{en} u_n$$

Coriolis

- **Coriolis term = planetary vorticity x tangential component of velocity,**
- **Absolute vorticity = relative + planetary vorticity;** $\eta = \zeta + f_o$
- **Combine momentum advection and Coriolis; nonlinear Coriolis force,**
- **Thuburn (2009) method for tangential velocity:**
  - Supports a stationary geostropic mode
  - Coriolis terms are energy conserving

Horizontal mixing

- **Horizontal mixing of momentum is also cast in terms of vorticity and kinetic energy,**
- **Assumes constant horizontal viscosity,**
- **Weakness in formulation, but we haven't yet seen negative consequences.**

$$\upsilon_h \left(\left[\nabla \delta_k \right]_e + \left[k \times \nabla \hat{\eta}_k \right]_e \right)$$

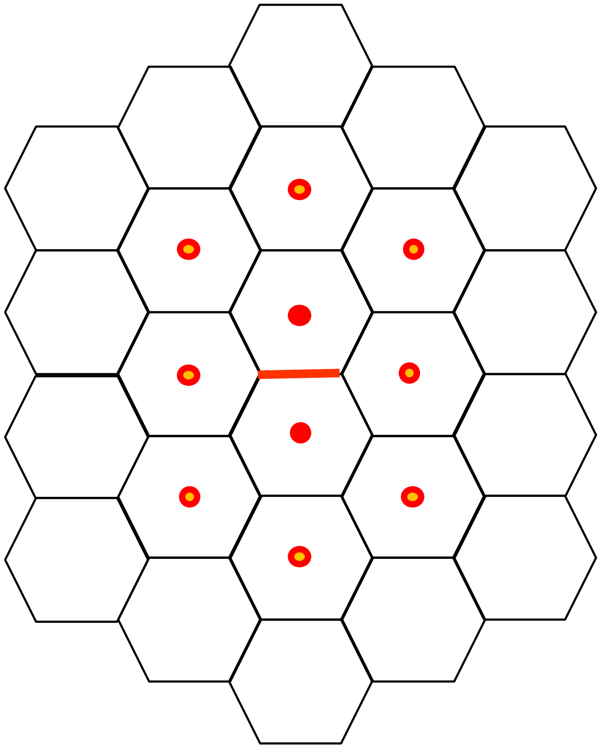
$\delta = \nabla \bullet \dot{u}$ Horizontal divergence

υ_h Horizontal viscosity

$\hat{\eta}$ Edge averaged absolute vorticity

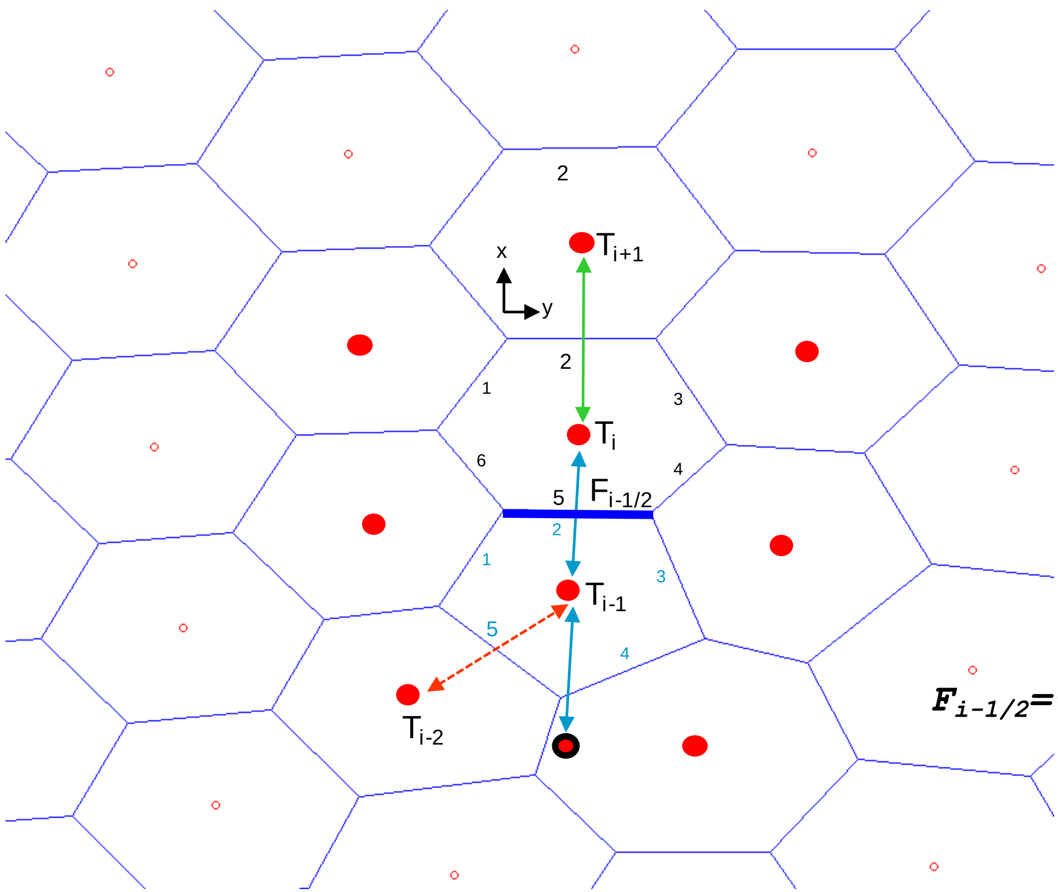
High order advection

- Use the MPAS algorithm - high order solution (3rd or 4th order via Skamarock and Gassmann, 2011).
- 3rd and 4th order schemes require second derivative at an edge.
- Uses a quadratic least squares polynomial to approximate second derivative required for high order schemes. $T = c_o + c_x x + c_y y + c_{xx} x^2 + c_{xy} xy + c_{yy} y^2$
- Use singular value decomposition (already in EMS) to pre-compute weighting matrix, then require vector operation to retrieve the second derivative value.
- Use FCT for monotonic solution.



Extension of least squares to QUICKEST

Least squares polynomial: $T=c_o+c_x x+c_y y+c_{xx}x^2+c_{xy}xy+c_{yy}y^2$



Regular grid mappings (direction 2)

Correct irregular grid mappings (direction 5)

Incorrect irregular grid mappings (direction 5)

Cell centres used in least squares fit

Least squares interpolated value

QUICKEST flux

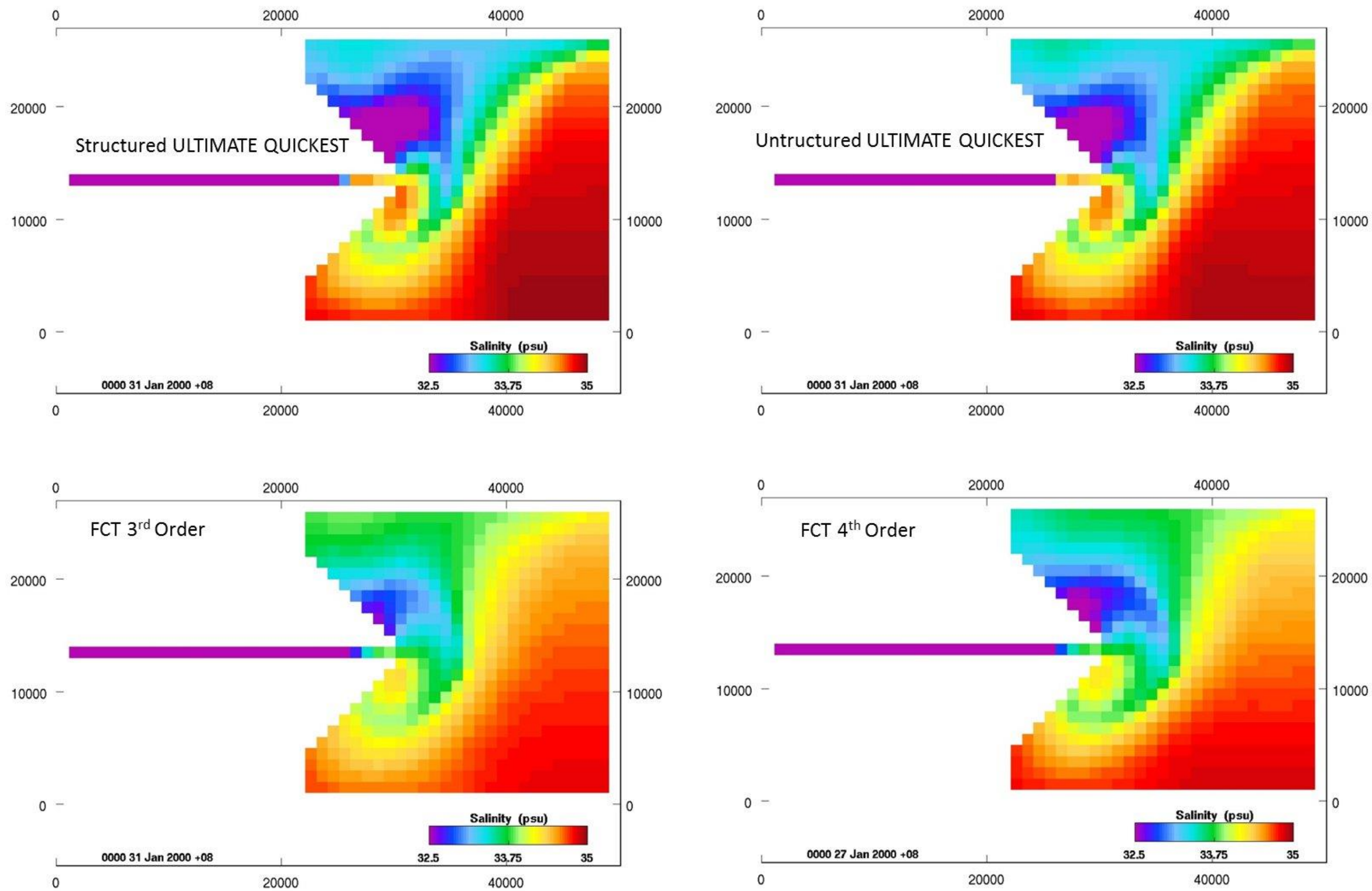
$$F_{i-1/2}=0.5(T_{i+1}+T_{i-1})-0.5q_{i-\frac{1}{2}}(T_i-T_{i-1})-\left[\tfrac{1}{6}-\tfrac{1}{6}q_{i-\frac{1}{2}}^2\right]CURV$$

$$q_{i-\frac{1}{2}}=u_i\Delta t/h$$

$$CURV=T_i-2T_{i-1}+T_{i-2} \quad for \;\; q_{i-\frac{1}{2}}>0$$

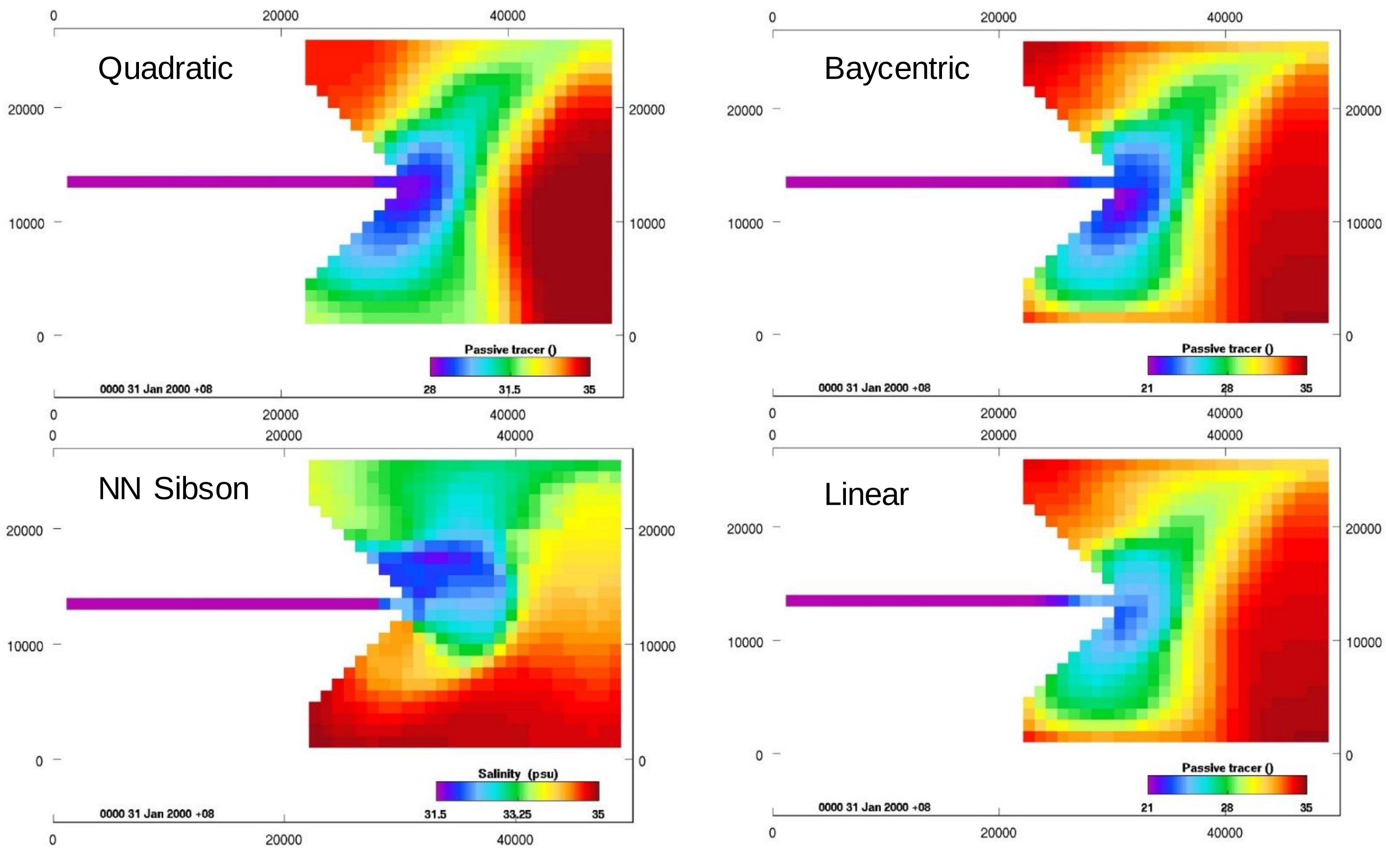
$$CURV=T_{i+1}-2T_i+T_{i-1} \quad for \;\; q_{i-\frac{1}{2}}<0$$

Tracer advection schemes



FCT: Flux corrected transport; (Zaleask (1979) J. Compt. Phys., 31, 335-362)
3rd or 4th order FCT (Skamarock and Gassmann, 2011, Mon. Wea. Rev., 139, 2962-2975).

Semi-Lagrange transport included for unstructured meshes

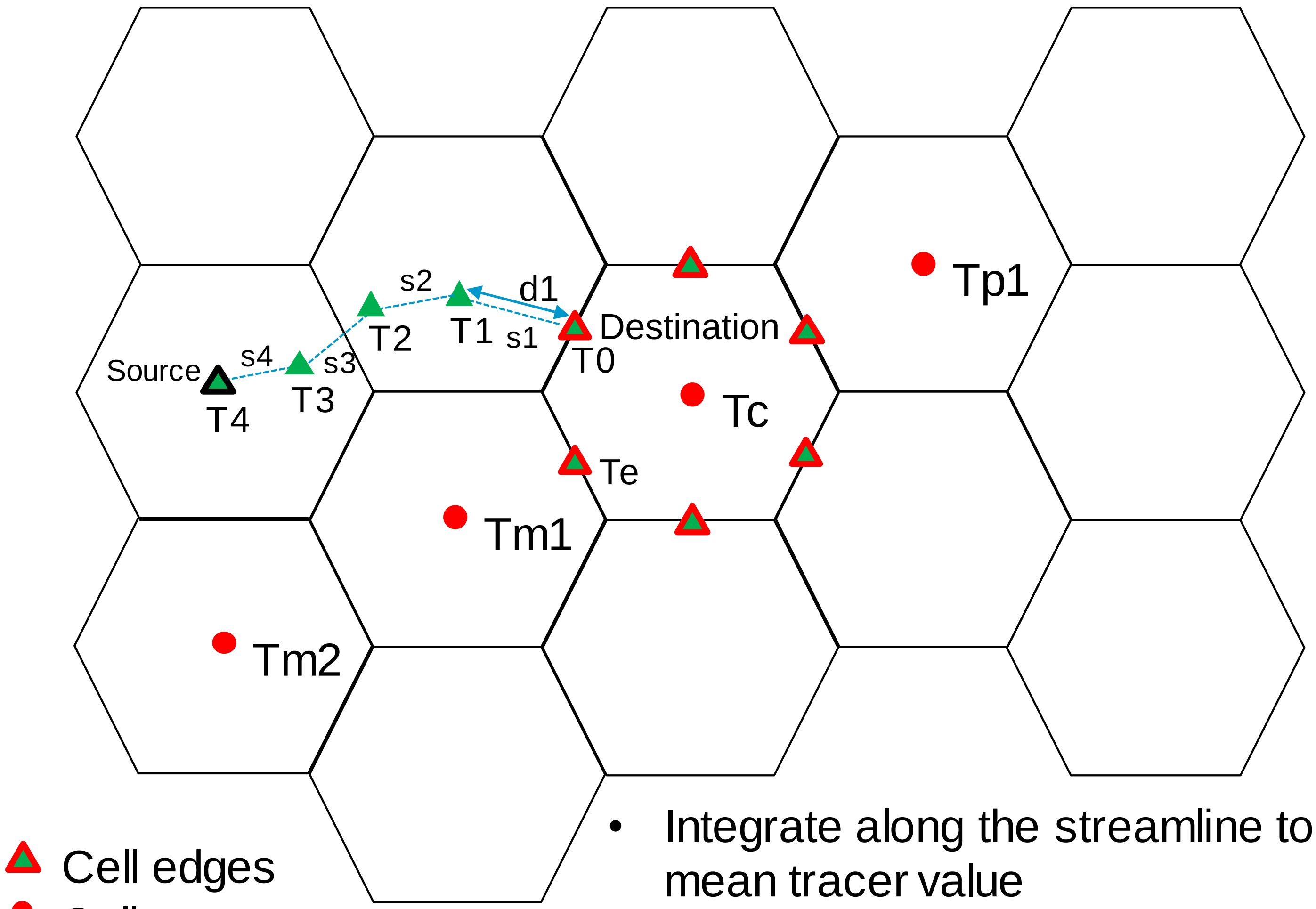


Semi-Lagrange schemes are unconditionally stable.

Interpolations available:

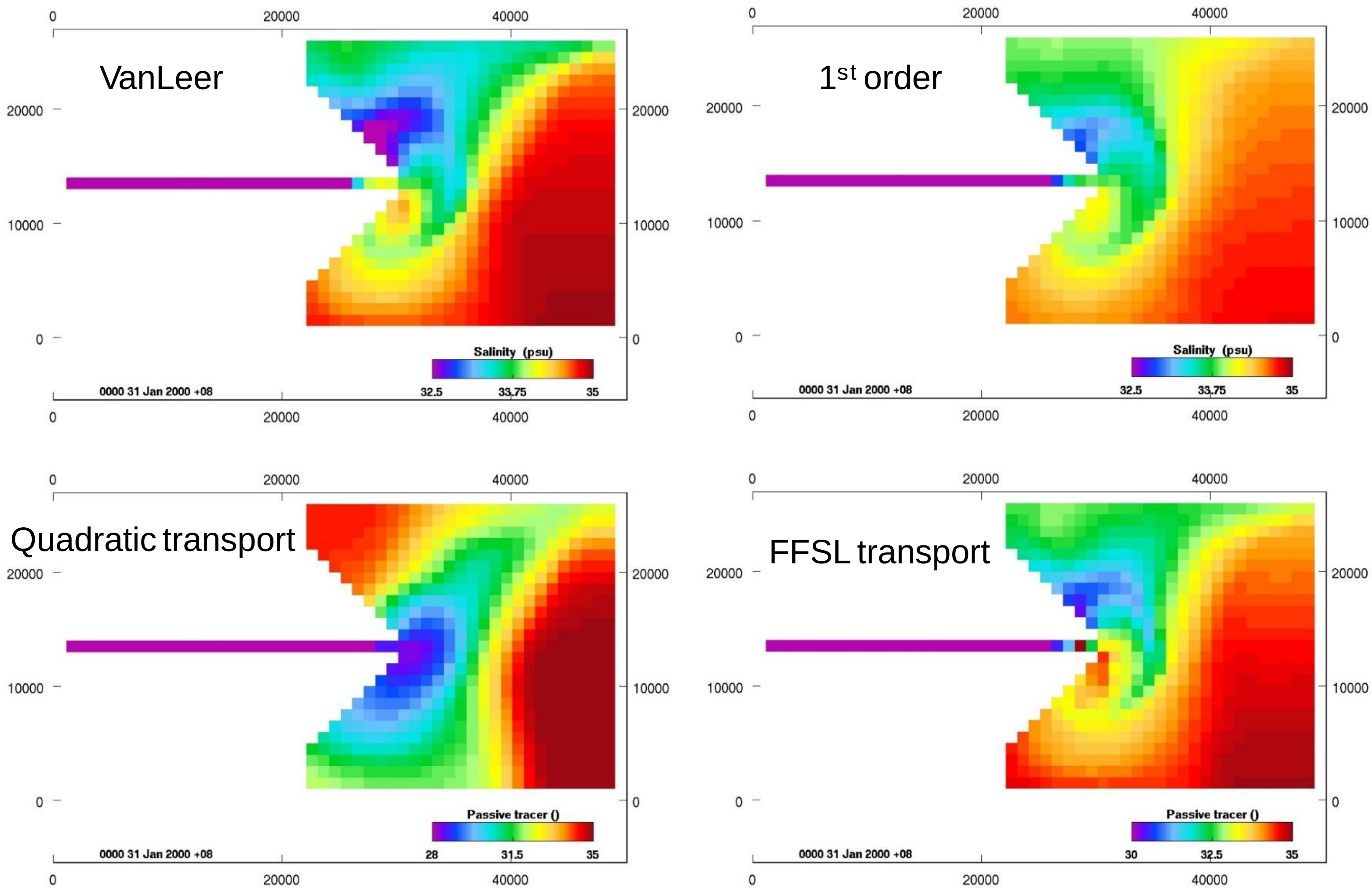
- Linear / baycentric linear - diffusive
- Cubic – non-monotonic
- Natural Neighbours – looks good but slow

Flux Form Semi-Lagrange schemes



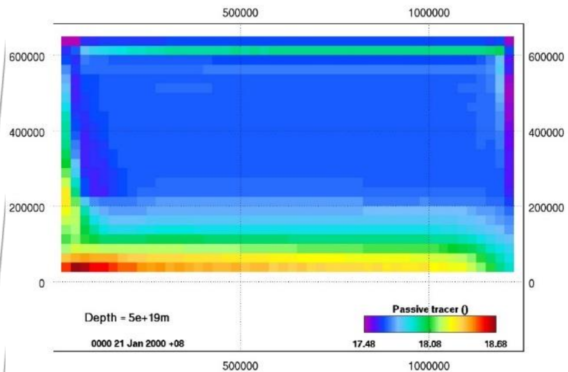
- Integrate along the streamline to obtain a mean tracer value
- Use these mean values in the advection scheme
- Requires a good limiter for monotinicity

FFSL comparisons

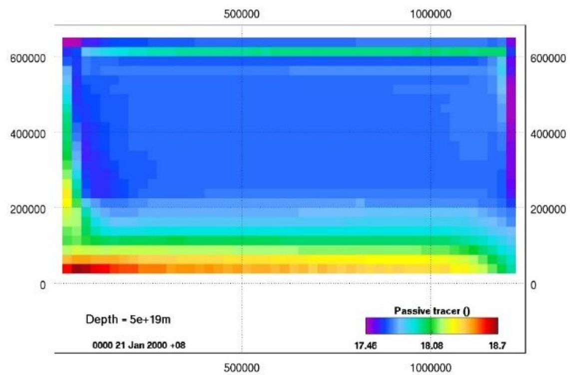


Large speedup while maintaining conservation

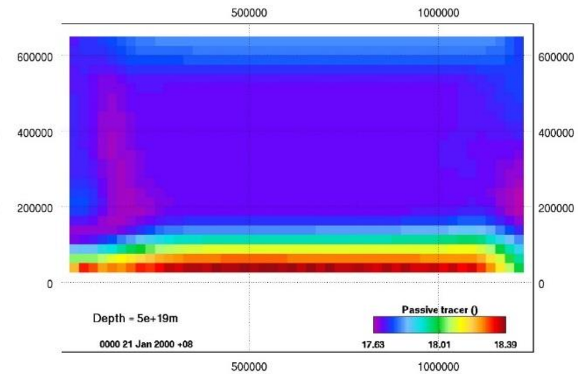
FFSL: DT = 1 hour : 12900:1

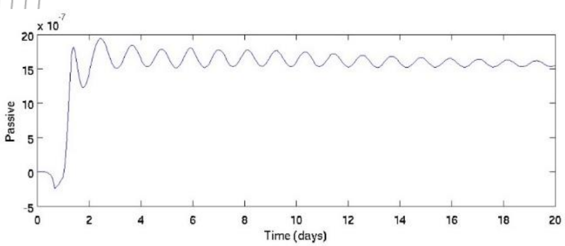


FFSL: DT = 12 hour : 152500:1

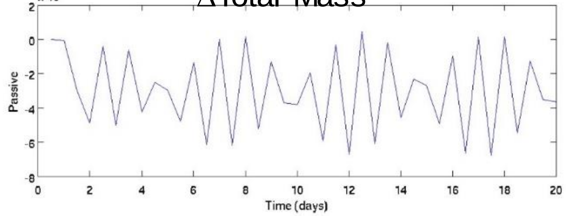


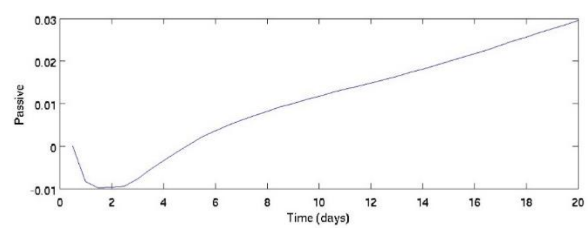
Lagrange DT = 12 hour : 339400:1



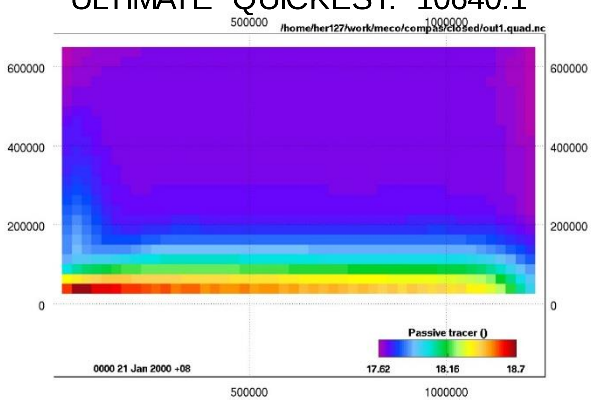


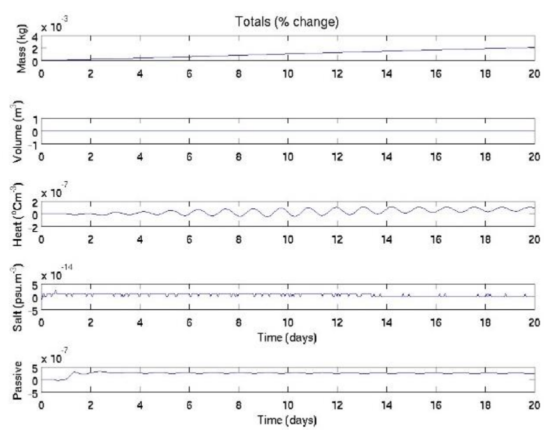
∆Total Mass





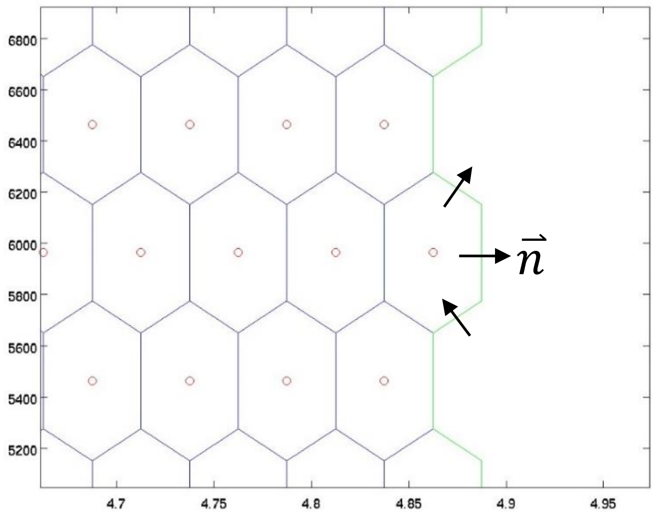
ULTIMATE QUICKEST: 10640:1

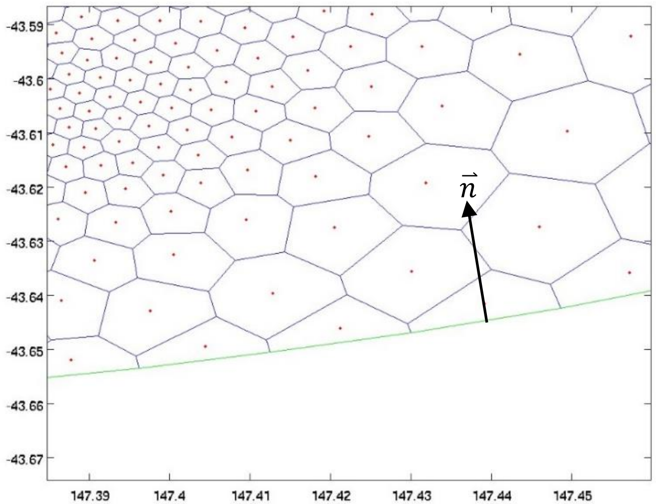




Open Boundaries

- **An OBC cell may be associated with multiple edges having different normal vector directions,**
- **The direction normal to the boundary is difficult to define,**
  - Radiation conditions may contain error
- **Full OBC suite is available,**
- **Flux adjusted Dirichlet condition (Herzfeld & Andrewartha, 2012) is well suited to finite volume,**
  - Flux adjustment is equally spread over cell edges.

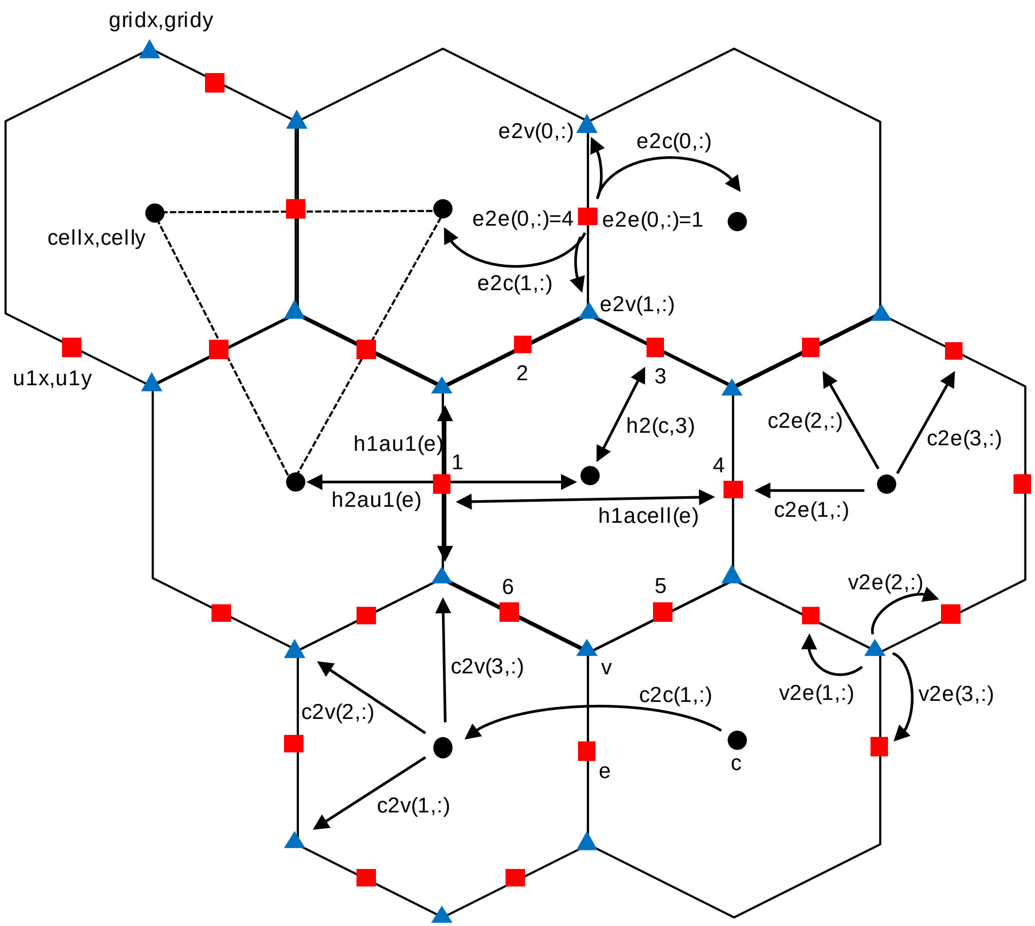




Integration into EMS

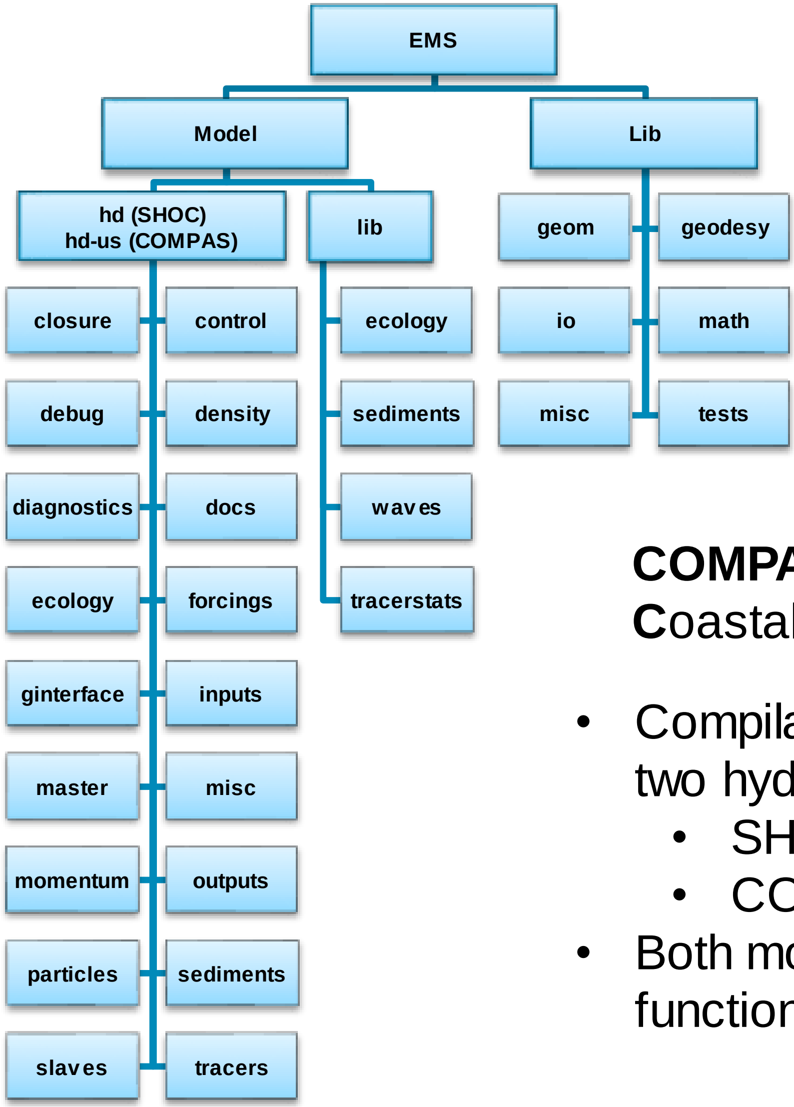
- **EMS contain a vast amount of supporting infrastructure (e.g. ROAM, open boundaries, BGC, sediment transport, waves, transport model, regional budget analysis…..)**
- **Two choices:**
  1. Adopt an alternative model and develop it to accept the EMS infrastructure,
  2. Retain EMS infrastructure and replace the hydrodynamic core.
- **Followed the second option:**
  - Less work
  - Seamless transition
    - Full backwards compatibility,
    - Same user operation, parameter files etc.
  - Retain development control

Unstructured system requires mappings



- Developed a generic unstructured system that supports all mappings required to run an unstructured algorithm
- (SHOC already uses an unstructured coordinate system)

Replace the EMS hydrodynamic core with MPAS-O

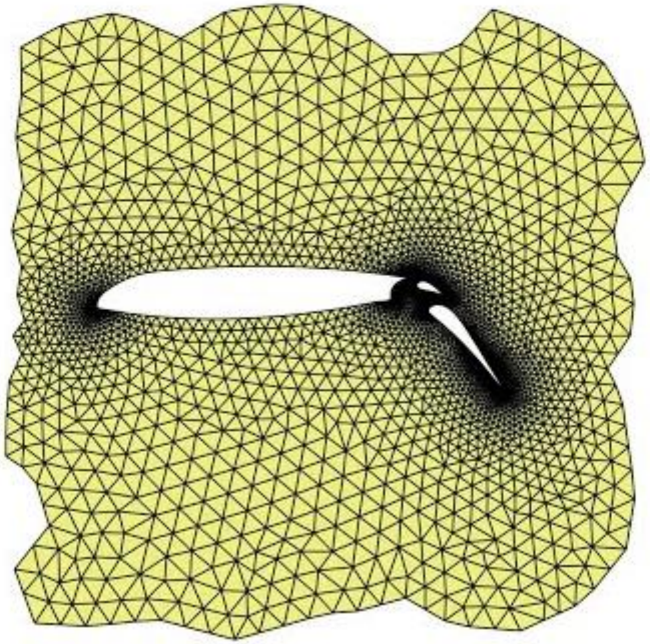


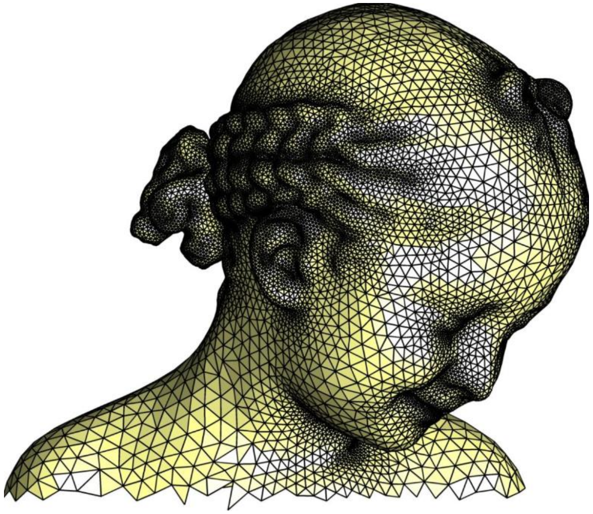
COMPAS:
Coastal **O**cean **M**odel **P**rediction **A**cross **S**cales

- Compilation of EMS provides two hydrodynamic models
  - SHOC (structured)
  - COMPAS (unstructured)
- Both models use library functions.

Grid generation

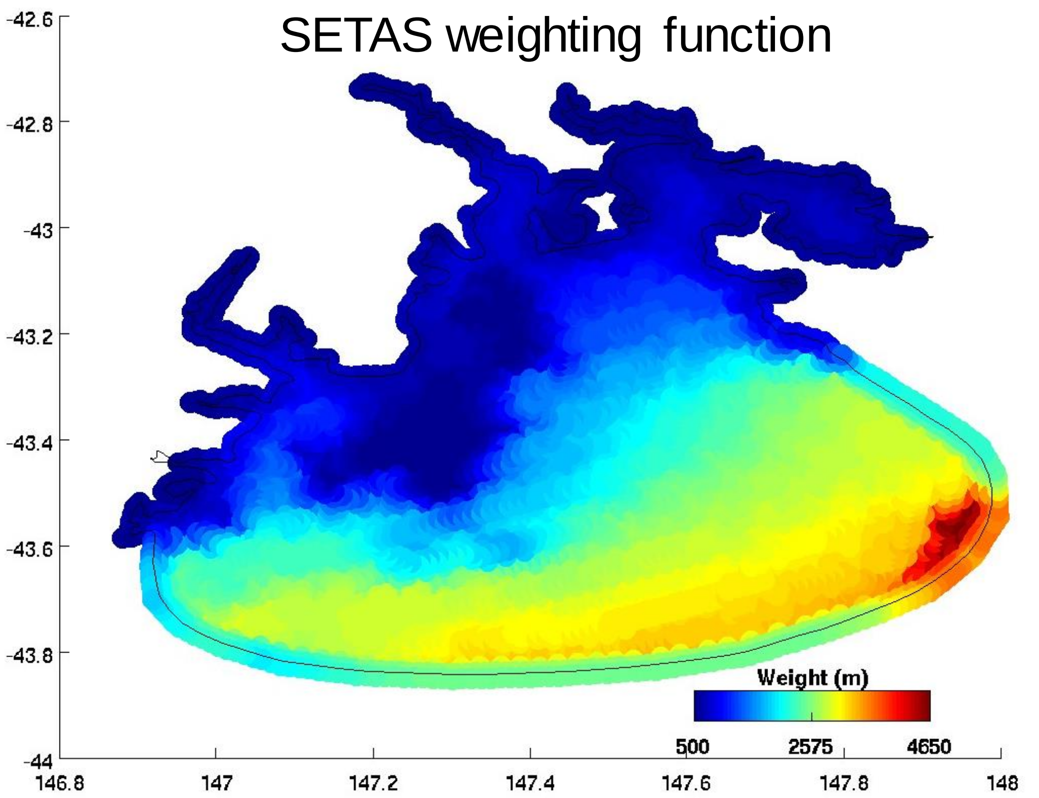
- **Use JIGSAW developed by Darren Engwirda (https://github.com/dengwirda/jigsaw)**
- **Designed to create MPAS compatible meshes**
  - Orthogonal Centroidal Voronoi Tesselation (CVT) meshes
- **Active collaboration with MPAS team (and now with CSIRO)**





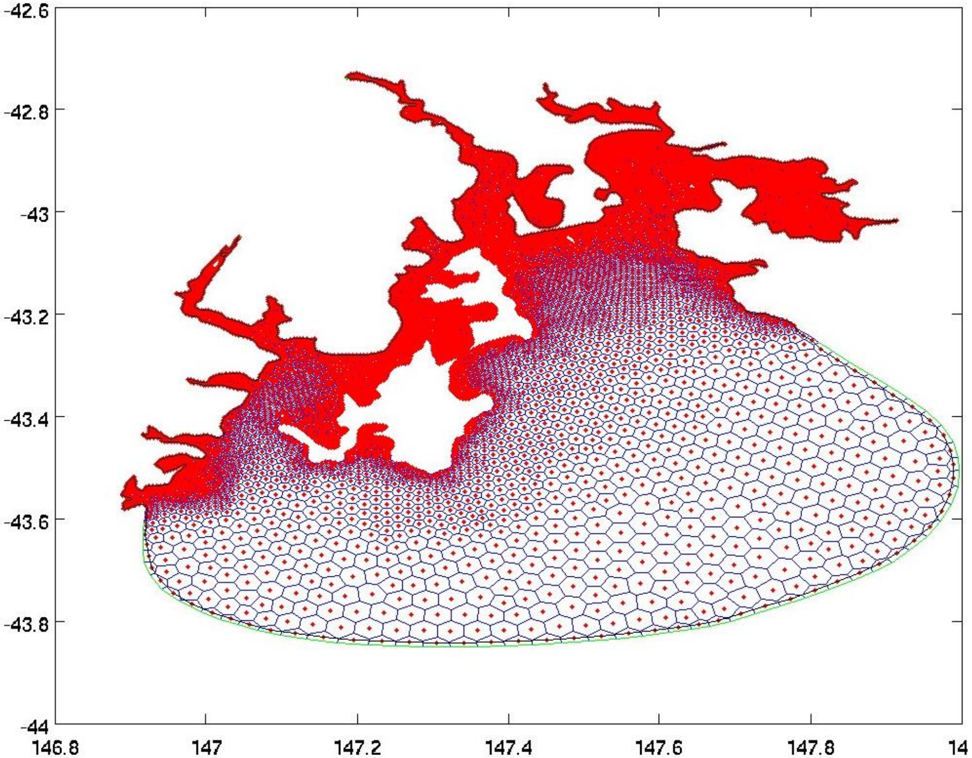
Use weighting function to define resolution

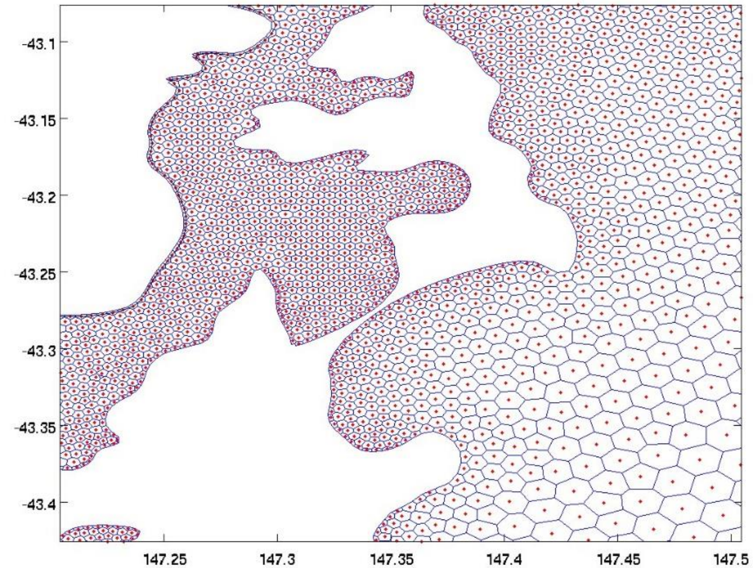
- **Determines what resolution is placed where in the grid,**
- **May be a function of:**
  - Bathymetry,
  - Distance from coast,
  - Turbulent kinetic energy
  - Tidal amplitude,
  - Salinity,
  - Arbitrary.

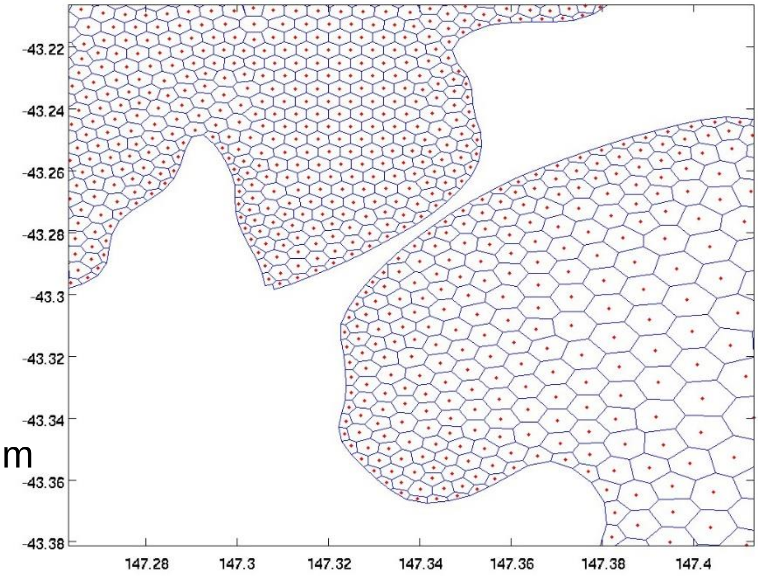


- **E.g. for bathymetry, specify (max / min) resolution for (min / max) depth, and map depths with linear, exponential, cosine etc. function.**

Example mesh: SE Tasmania



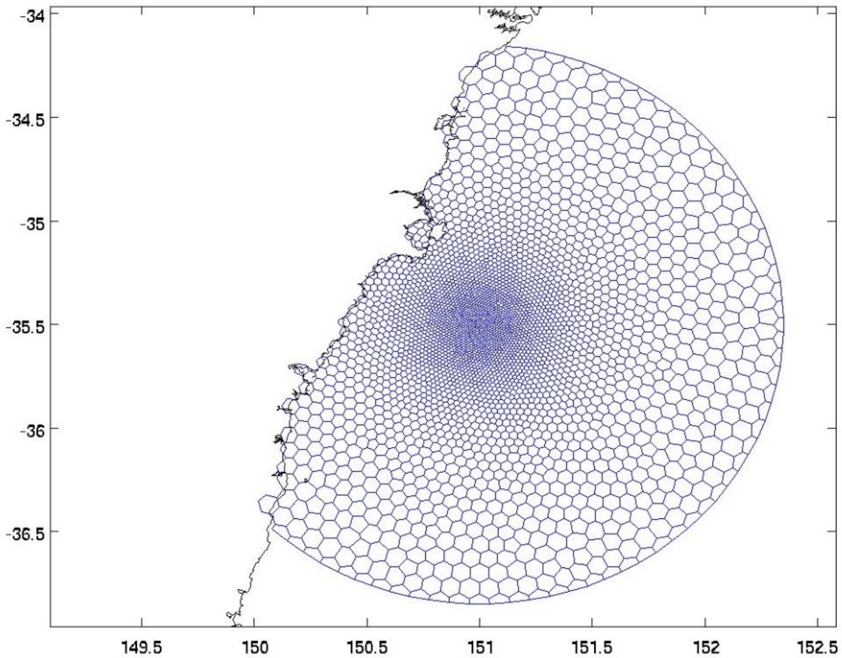




10072 cell centres
Mean horizontal edge length = 673.40 m
Mean horizontal distance between centres = 385.17 m
Minimum horizontal distance between centres = 116.94 m
Maximum horizontal distance between centres = 5039.33 m
Mean cell area = 0.62 km^2
STORM 74262 cells, 12:1

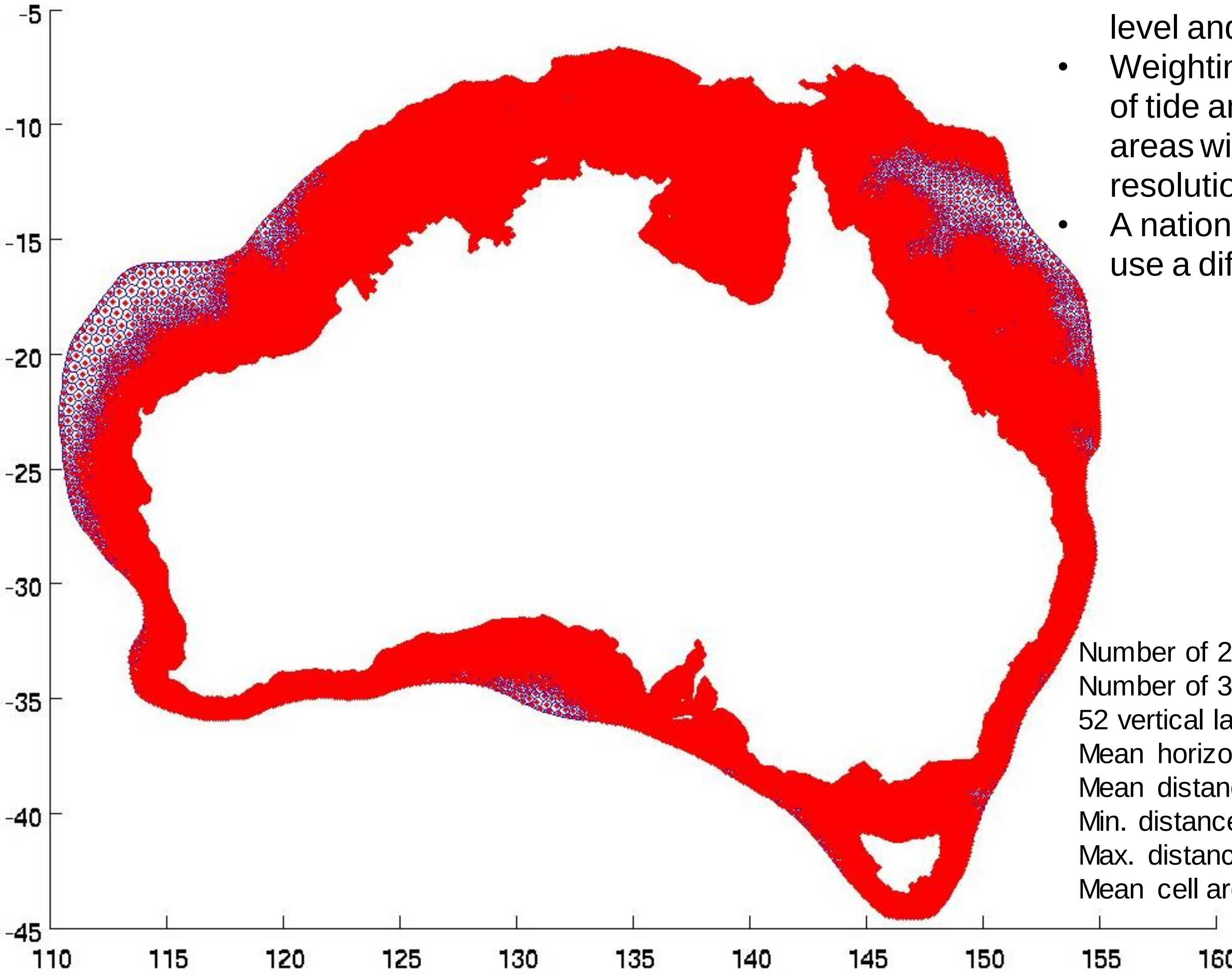
Example mesh: Hi-resolution automated meshes

- **Weighting function is Gaussian centered on a defined location,**
- **Creates circular mesh with maximum resolution at centre, minimum resolution at edges,**
- **Minimizes OBC specification error by using 1:1 boundary ratios.**



Mesh over EAC region, with coastal masking.
Maximum resolution ~800 m
Minimum resolution ~11 km
Mean resolution ~1.8 km

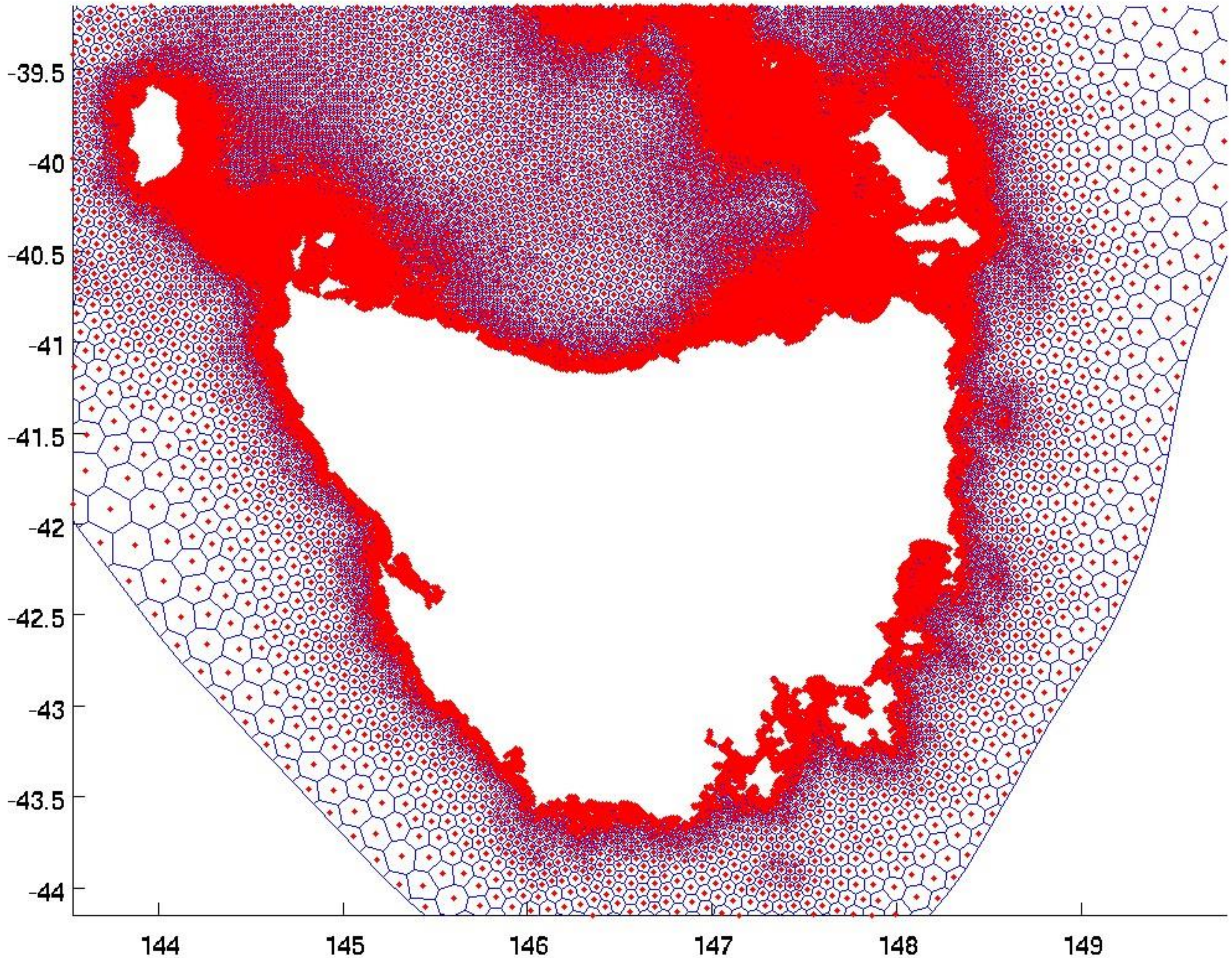
Example mesh: Australian shelf (ARENA)



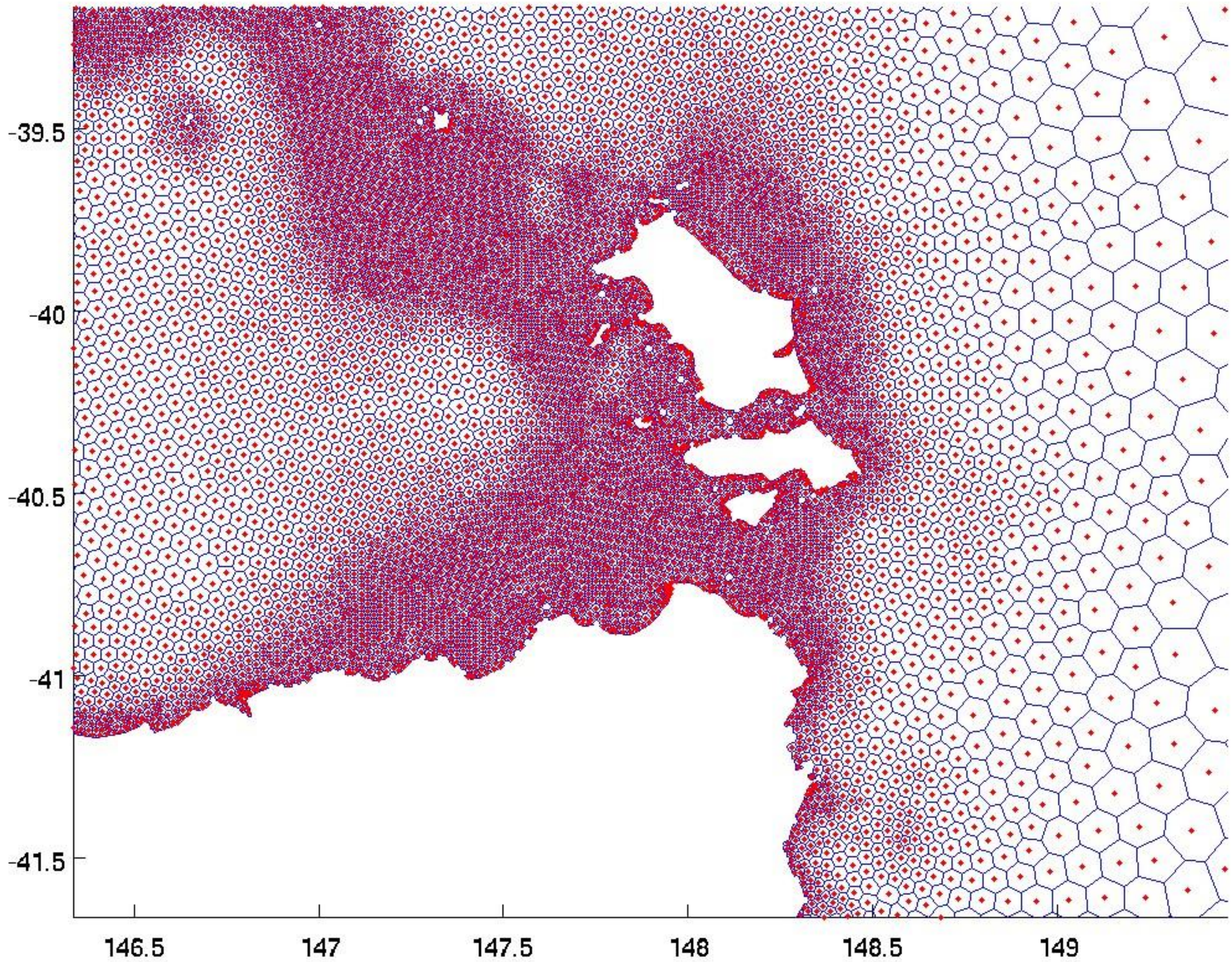
- ARENA project is to deliver tidal sea level and currents around Australia,
- Weighting function is a combination of tide and bathymetry (e.g. shallow areas with high amplitude get high resolution),
- A national 3D model would probably use a different weighting function.

Number of 2D wet cells = 212686
Number of 3D wet cells = 2322647
52 vertical layers
Mean horizontal edge length = 4162.61 m
Mean distance between centres = 2394.27 m
Min. distance between centres = 445.25 m
Max. distance between centres = 54501.19 m
Mean cell area = 23.23 km^2

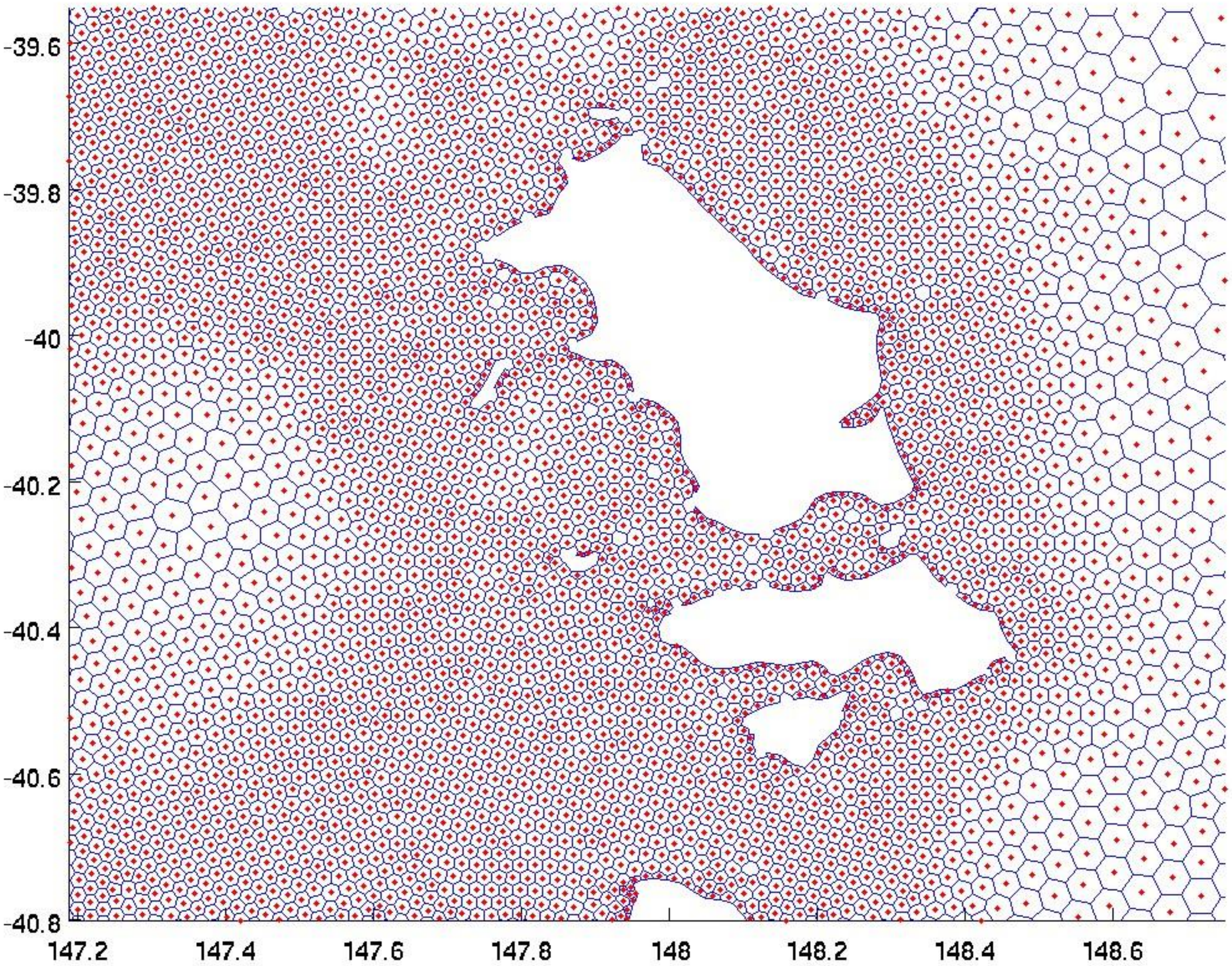
Seamless resolution transition



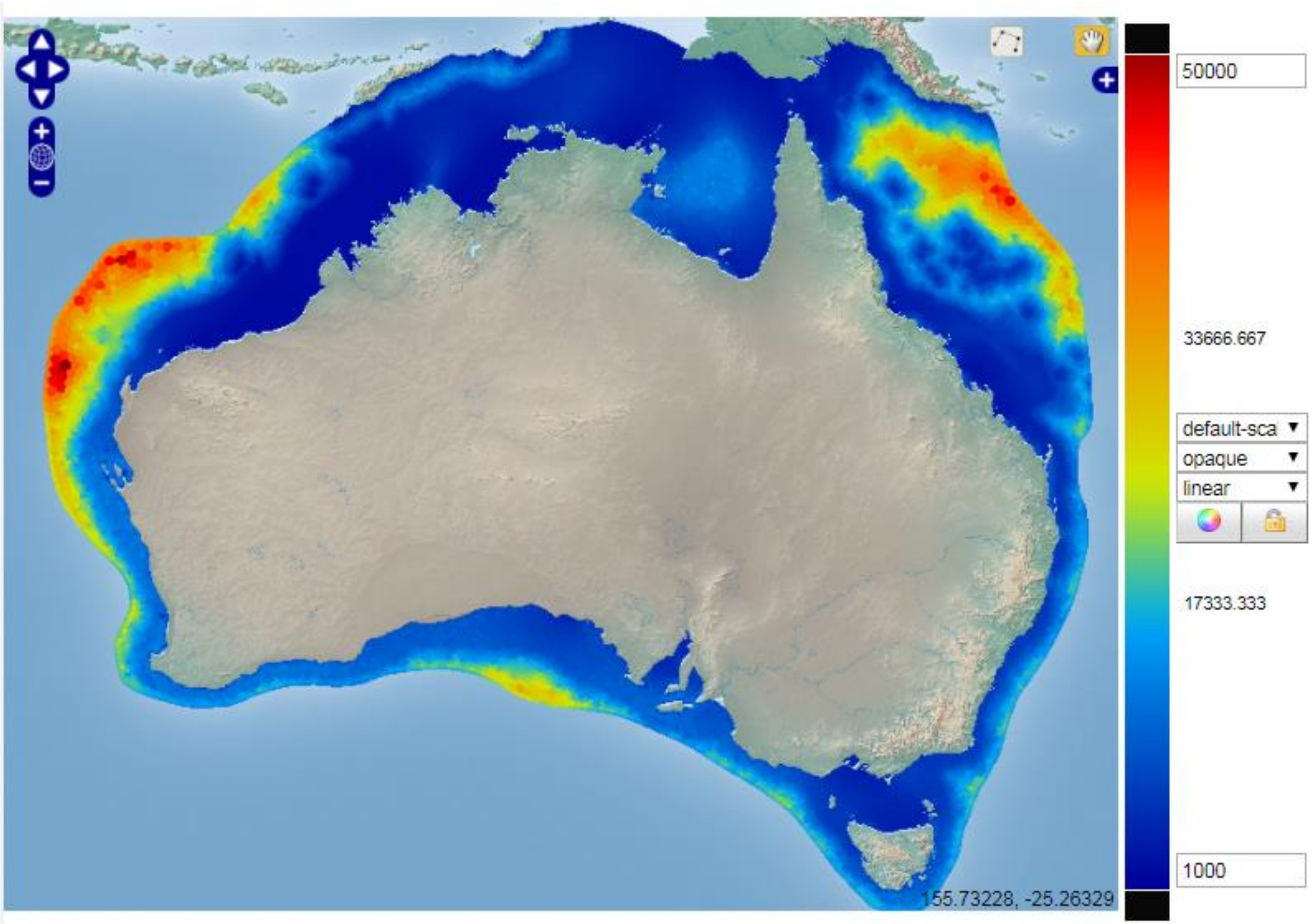
Precisely prescribe areas of high resolution



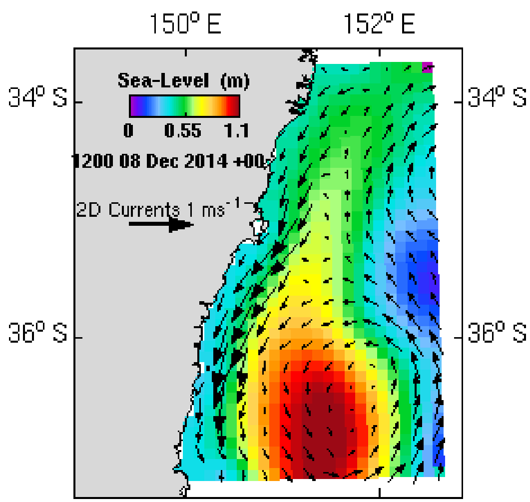
Flinders Island ~4 km

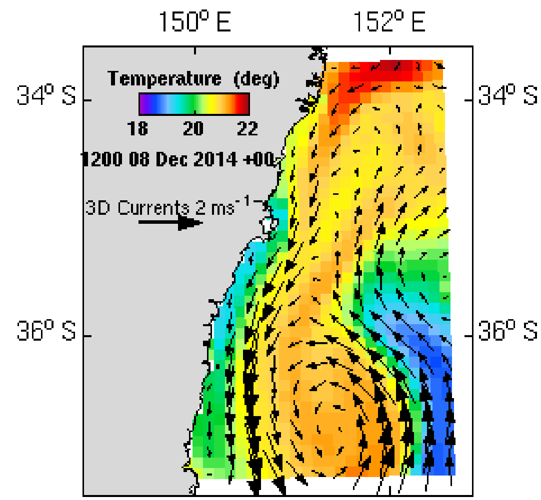


Resolution map



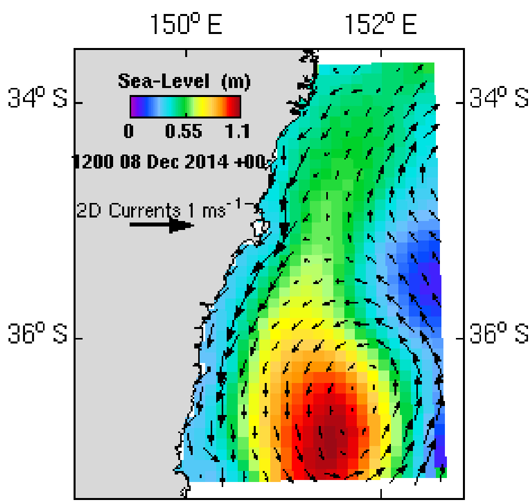
ROAM: COMPAS vs SHOC for EAC, Dec 2014

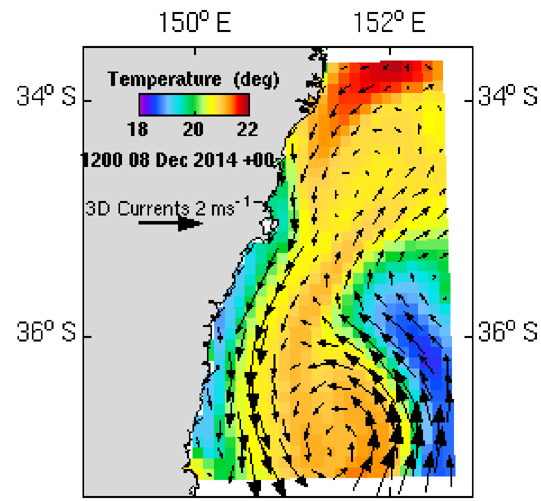




SHOC

Sea level and 2D depth averaged currents (left) and surface temperature and 3D surface currents (right) for EAC region. Model is 10 km resolution.

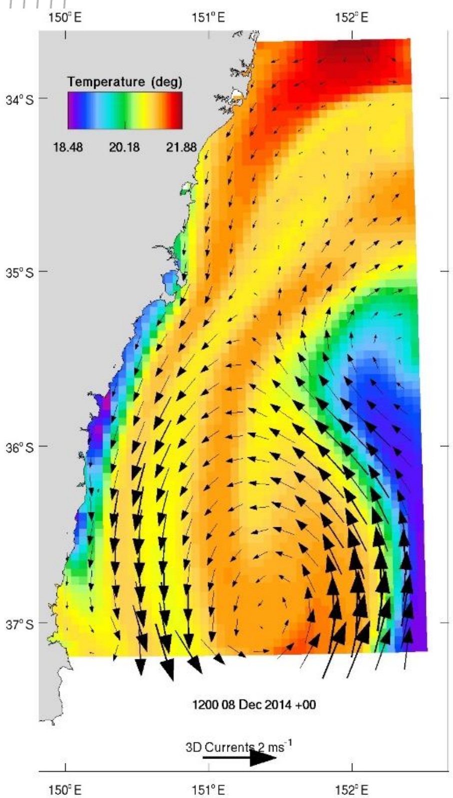




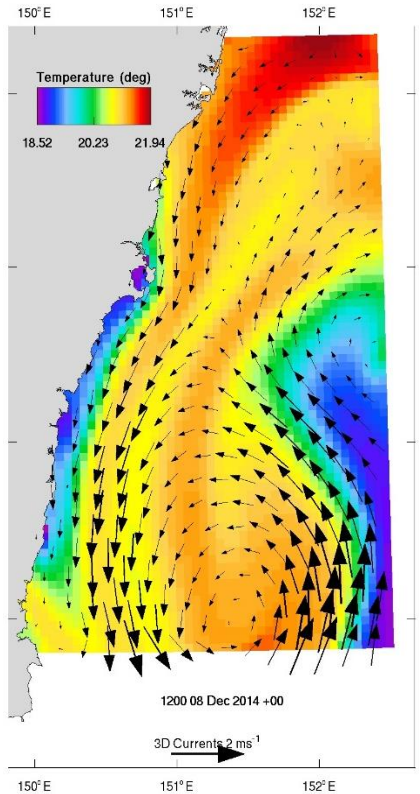
COMPAS

Comparison to hex meshes, EAC, Dec 2014

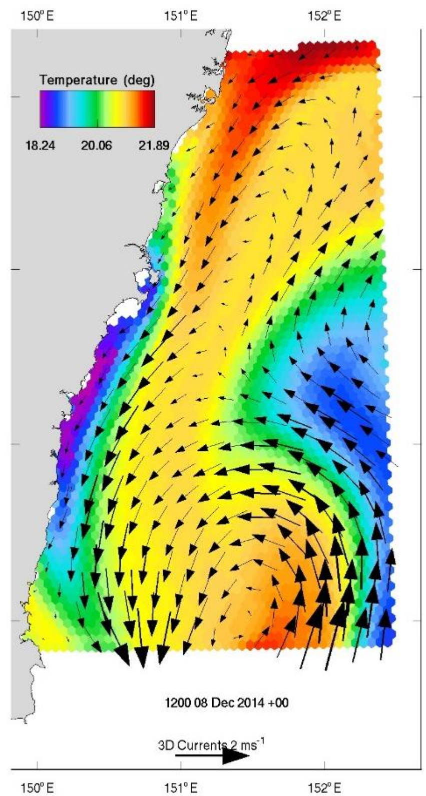
Surface temperature and currents. Resolution ~5km.



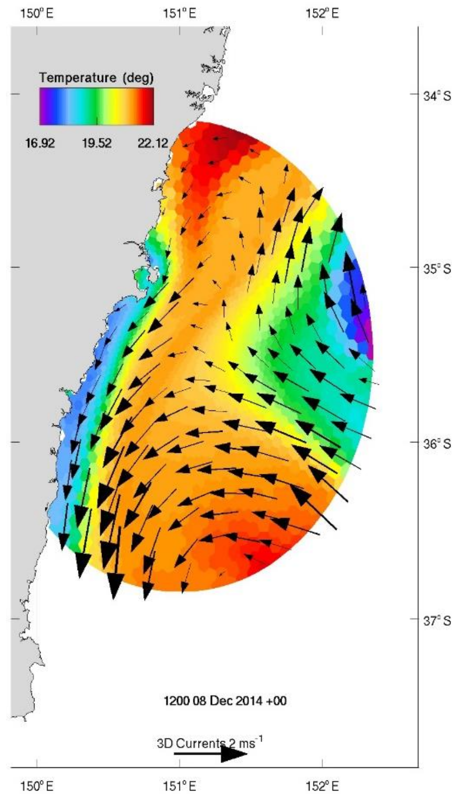
Structured quad



COMPAS quad

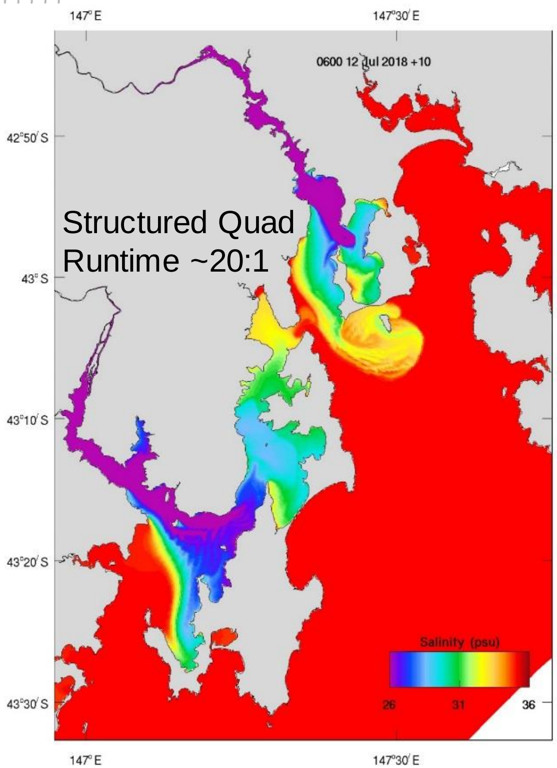


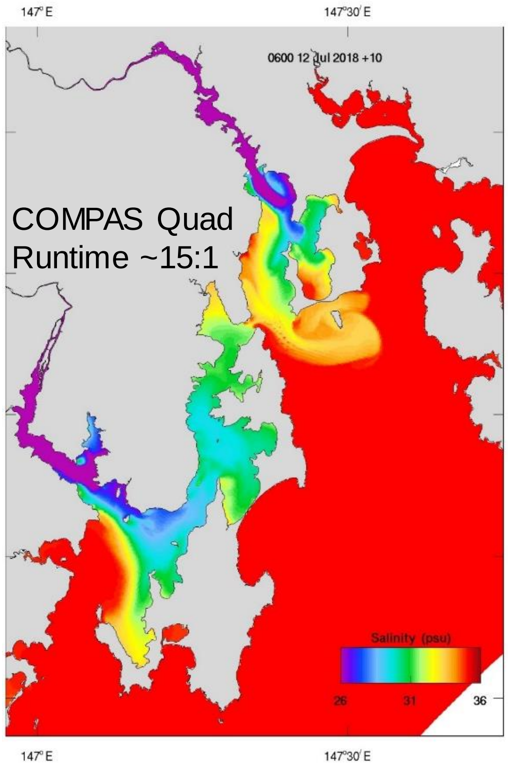
COMPAS hex

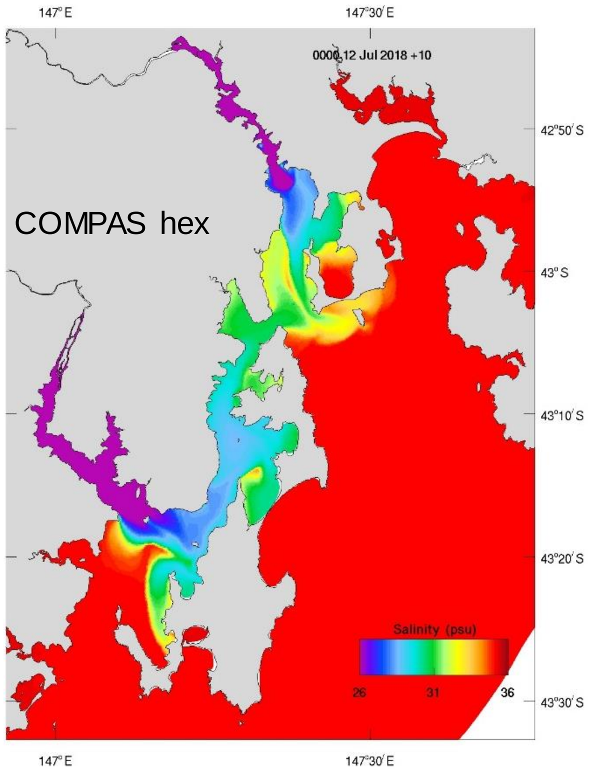


COMPAS variable

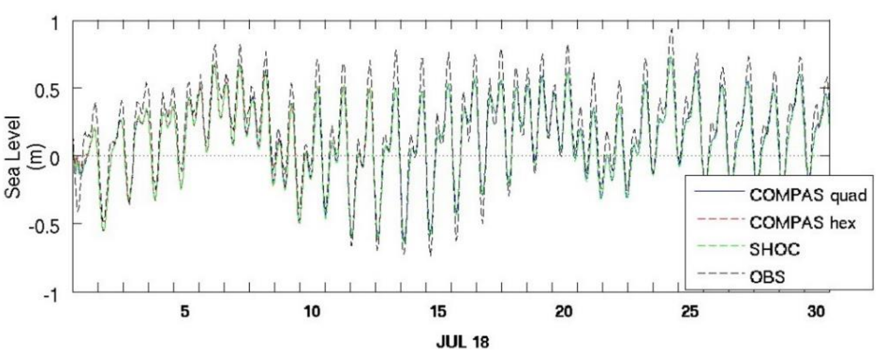
South East Tasmania (CTAS)







Sea level at Hobart



QUAD / Hex

2D wet cells = 54404 / 34992

3D wet cells = 1014963 / 414185

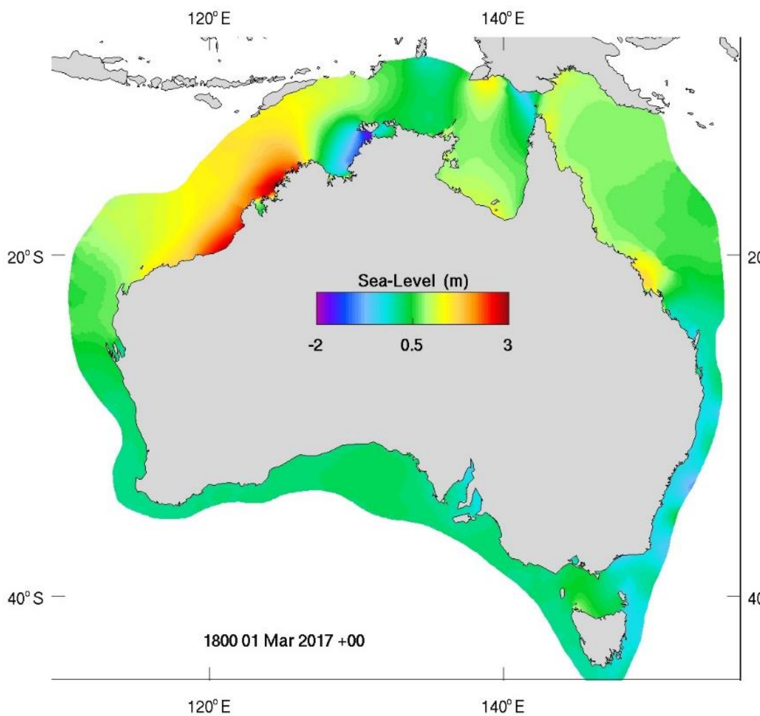
Mean horizontal edge length = 252.90 / 198.67 m

Mean horizontal distance between centres = 252.75 / 114.28 m

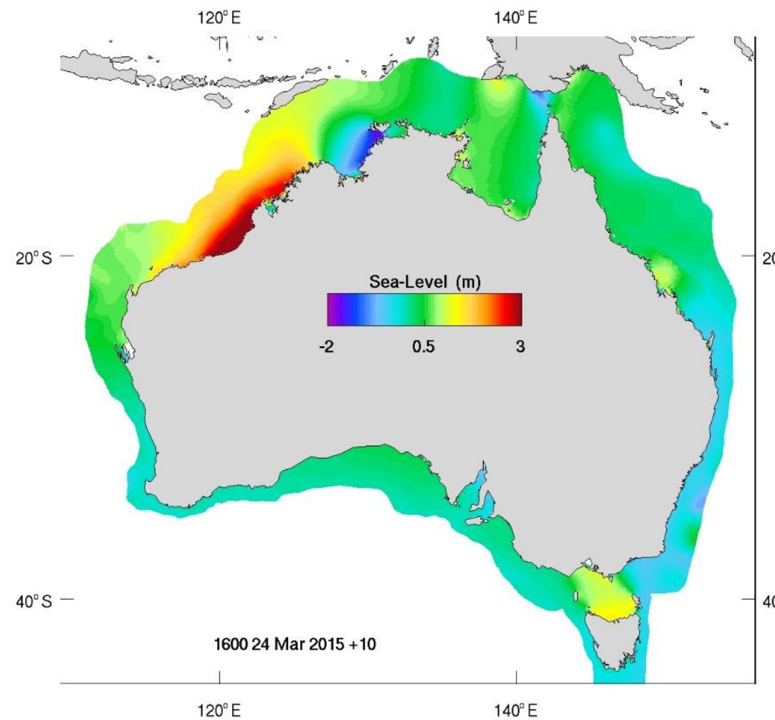
Minimum horizontal distance between centres = 101.10 / 40.73 m

Maximum horizontal distance between centres = 992.22 / 3384.05 m

ARENA



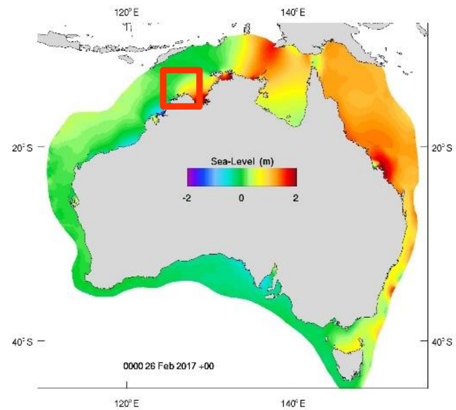
COMPAS



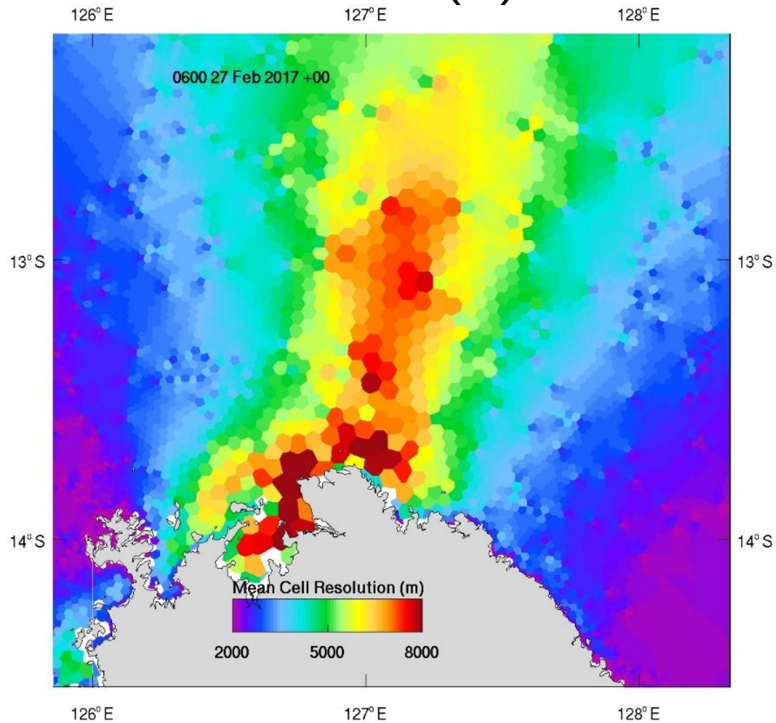
Ribbon Model (equivalent phase of tide)

- Running the ARENA grid in full 3D,
- Model is very slow (0.6:1 on 1 PE),
- Tidal model to be run in 2D mode for neap-spring cycle,
- Probably can increase resolution by factor of 4.

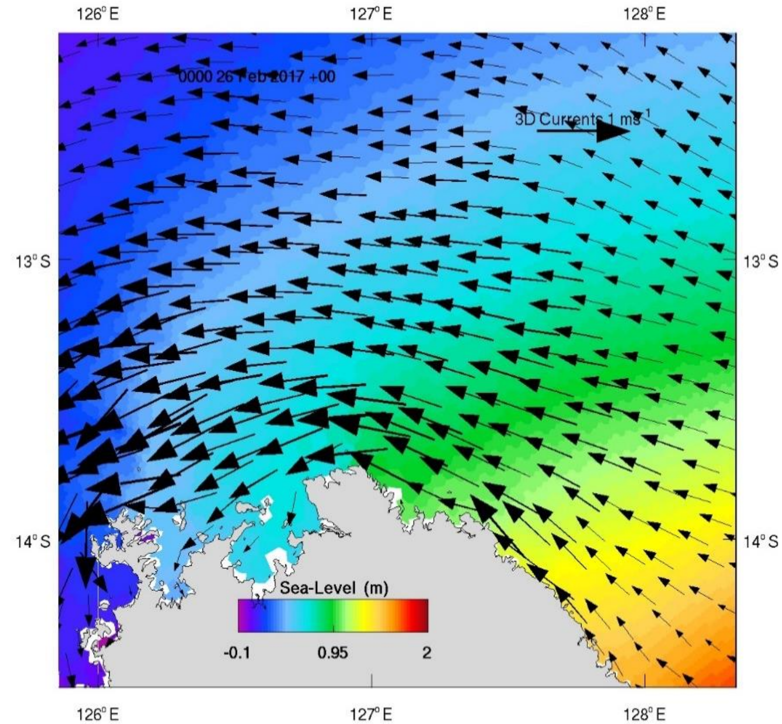
Resolution transition effects



Resolution (m)

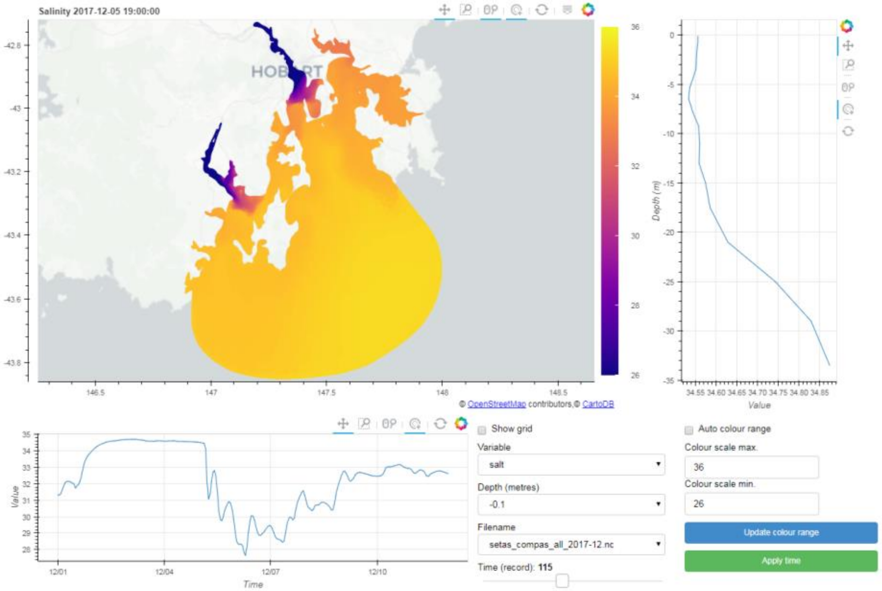


Sea level & surface currents



Visualisation: UGRID

- **CF compliant netCDF standard developed (CF-1.6 UGRID-1.0) (https://github.com/ugrid-conventions/ugrid-conventions)**
- **UGRID conventions described under http://ugrid-conventions.github.io/ugrid-conventions/**
- **We use 3D layered mesh topology**
- **UGRID compatible netCDF can be visualized by Godiva 3**
- **Can use ParaView with UGRID reader plugin**
- **Developing in-house tools using python / bokeh (web based)**



Status

- **COMPAS is a viable unstructured model.**
- **Examples documented on:** **https://research.csiro.au/cem/software/ems/hydro/unstructured-compas/**
  - Quad and hex models functional in simple test pools, closed basin (wind forced), test estuary (wind, tidal, river forced).
  - Quad and hex models functional in regular ROAM-like domain (EAC) with full forcing.
  - Quad model functional in irregular domain (SE Tas) with full forcing.
- **EMS grid generation coupled to JIGSAW inline.**
- **Beta version available on GitHub:**

https://github.com/csiro-coasts/EMS

Thank you

- **Coastal Development and Management**
  - Mike Herzfeld

- **t** +61 3 6232 5167
- **e** mike.herzfelde@csiro.au
- **w** http://www.emg.cmar.csiro.au/www/en/emg.html